O-28

Tunneling Problems of Excitations in Spin-1 BEC

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Contents

- Introduction of tunneling problem of excitations in BEC
- Ground state and low lying excitations in spin-1 BEC
- Main results (Ferromagnetic state)
- Discussion
- How about Polar state?

Perfect transmission of excitations in a scalar BEC

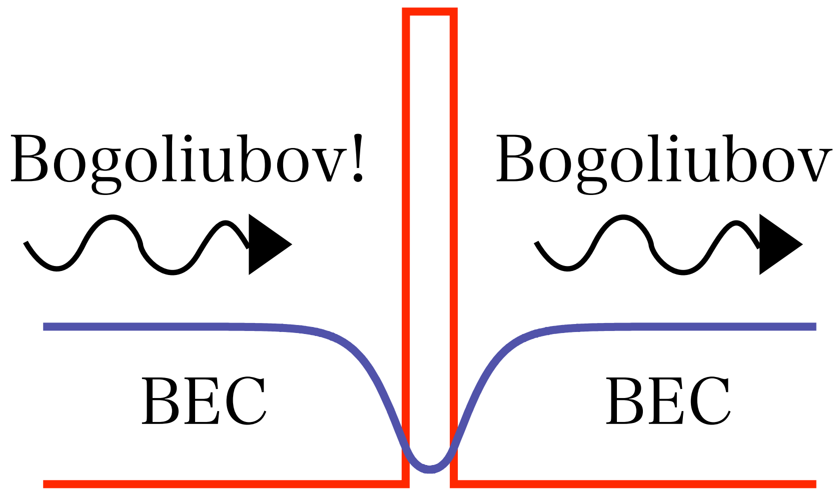
- Gross-Pitaevskii equation

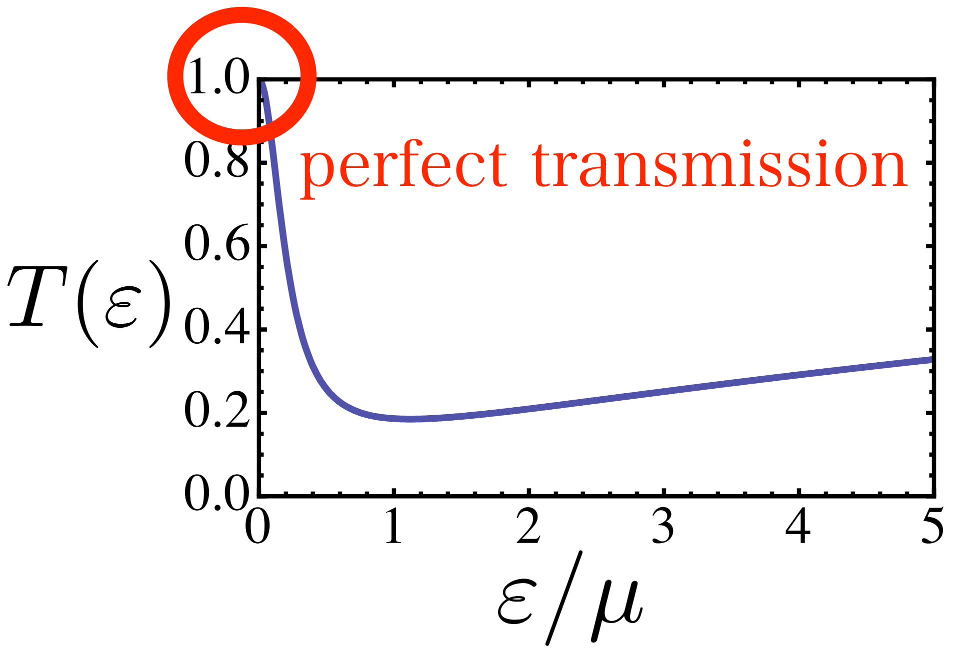
$$[\hat{H}_0(\mathbf{r}) + g|\Phi_0(\mathbf{r})|^2]\Phi_0(\mathbf{r}) = 0 \qquad \hat{H}_0(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) - \mu$$

- Bogoliubov equation

$$\begin{pmatrix} \hat{H}_0 + 2g|\Phi(\mathbf{r})|^2 & -g[\Phi_0(\mathbf{r})]^2 \\ g[\Phi_0^*(\mathbf{r})]^2 & -\hat{H}_0 - 2g|\Phi_0(\mathbf{r})|^2 \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \varepsilon_i \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

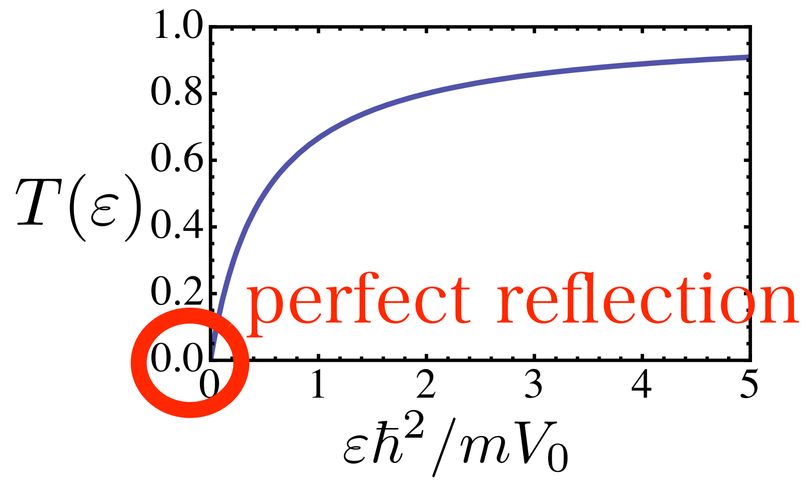
Kovrizhin et al.(2000) Kagan et al. (2003)



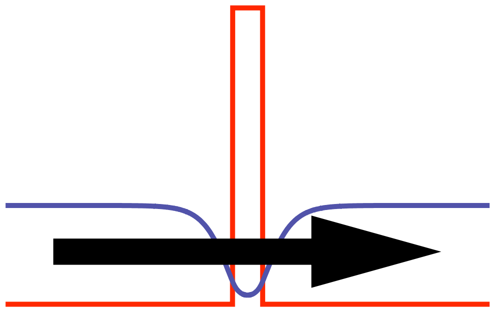


Why so important?

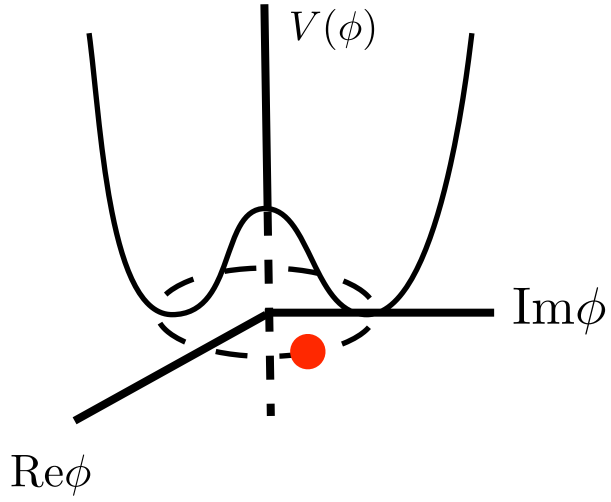
- contrast to single particle tunneling in quantum mechanics



- not a supercurrent flow but excitations



- clue for understanding the low lying excitation of BEC



- relation with Tomonaga-Luttinger liquids

recent study

- mechanism of anomalous tunneling

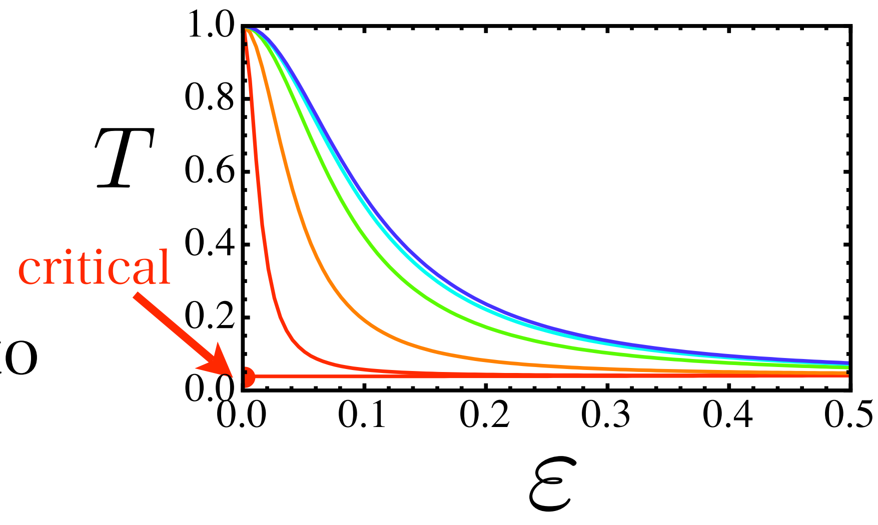
  Danshita et al., Kato et al., Tsuchiya et al., and Ohashi et al.

- under supercurrent

  Danshita et al., Ohashi et al.

  on the critical current

  Danshita et al., and Takahashi and Kato



- at finite temperatures

  Nishiwaki and Kato

- reflection and refraction

  Watabe and Kato

- relation with Tomonaga-Luttinger liquid

  Watabe and Kato

spin-1 BEC

- scalar BEC

  magnetic trap

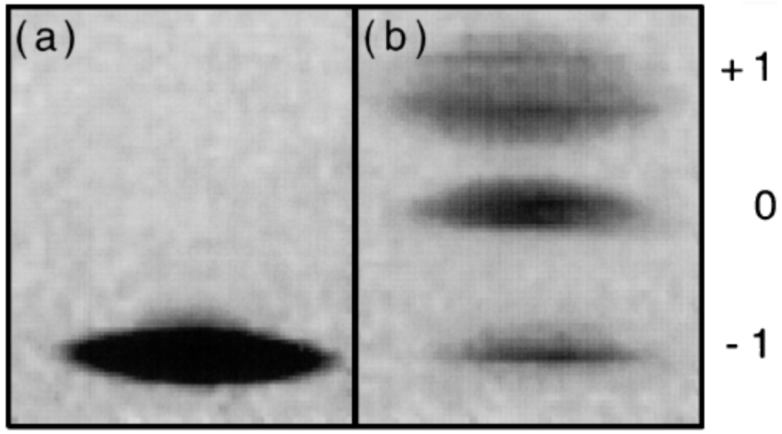
  internal degrees of freedom are frozen

  order parameter is scalar

- spinor BEC

  optical trap

  spin of alkali atoms are free



D. Stamper-Kurn, et.al., PRL (1998) ^{23}Na

- spin-1 BEC

  ^{23}Na polar state

  ^{87}Rb ferromagnetic state

Motivation

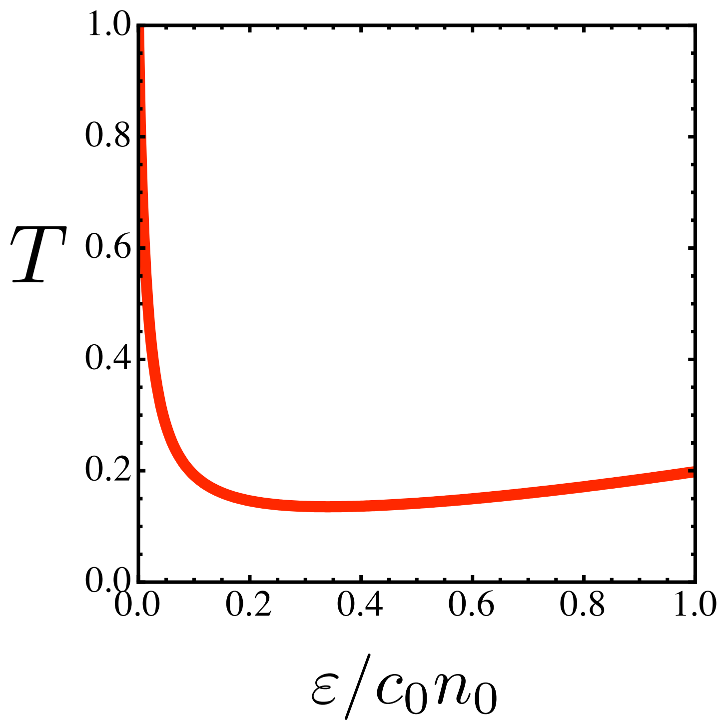
- How is excitations scattered against the potential barrier in the spinor BEC?

^{87}Rb ferromagnetic state

^{23}Na polar state

Bogoliubov excitation, Magnon

- Excitations but imcompressible magnon mode in ferromagnetic state experience perfect transmission in the long wavelength limit.



Hamiltonian and Ground State

Ohmi and Machida (1998), Tin-Lun Ho (1998)

kinetic energy, potential energy, Hartree, spin-exchange

- $$E = \int d\mathbf{r} \left[\sum_i \frac{\hbar^2}{2m} |\nabla \Phi_i(\mathbf{r})|^2 + U(\mathbf{r}) n(\mathbf{r}) + \frac{c_0}{2} n^2(\mathbf{r}) + \frac{c_2}{2} n^2(\mathbf{r}) |\langle \mathbf{F}(\mathbf{r}) \rangle|^2 \right]$$

density $n(\mathbf{r}) = \sum_i |\Phi_i(\mathbf{r}, t)|^2$

local spin density $\langle \mathbf{F}(\mathbf{r}) \rangle = \frac{1}{n(\mathbf{r})} \sum_{ij} \Phi_i^*(\mathbf{r}) \mathbf{F}_{ij} \Phi_j(\mathbf{r})$

- ferromagnetic state $c_2 < 0$

$$|\langle \mathbf{F} \rangle| = 1 \qquad \begin{pmatrix} \Phi_{+1} \\ \Phi_0 \\ \Phi_{-1} \end{pmatrix} = \sqrt{n} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- polar state $c_2 > 0$

$$|\langle \mathbf{F} \rangle| = 0 \qquad \begin{pmatrix} \Phi_{+1} \\ \Phi_0 \\ \Phi_{-1} \end{pmatrix} = \sqrt{n} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Ferromagnetic State

^{87}Rb

$c_2 < 0$

$|\langle \mathbf{F} \rangle| = 1$

$$\begin{pmatrix} \Phi_{+1} \\ \Phi_0 \\ \Phi_{-1} \end{pmatrix} = \sqrt{n} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$

- fluctuation of order parameter $\delta\theta, \ \delta n$ m=+1 ⟶ m=+1

  Bogoliubov excitation

  $$E = \sqrt{\varepsilon[\varepsilon + 2(c_0 + c_2)n_0]}$$

- spin fluctuation δM_- m=+1 ⟶ m=0

  magnon mode

  $$E = \varepsilon$$

- quadratic spin fluctuation δM_-^2 m=+1 ⟶ m=-1

  magnon mode

  $$E = \varepsilon + 2|c_2|n_0$$

Results : Ferromagnetic state

- fluctuation of order parameter

  m=+1 ⟶ m=+1

  $$E = \sqrt{\varepsilon[\varepsilon + 2(c_0 + c_2)n_0]}$$

- spin fluctuation

  m=+1 ⟶ m=0

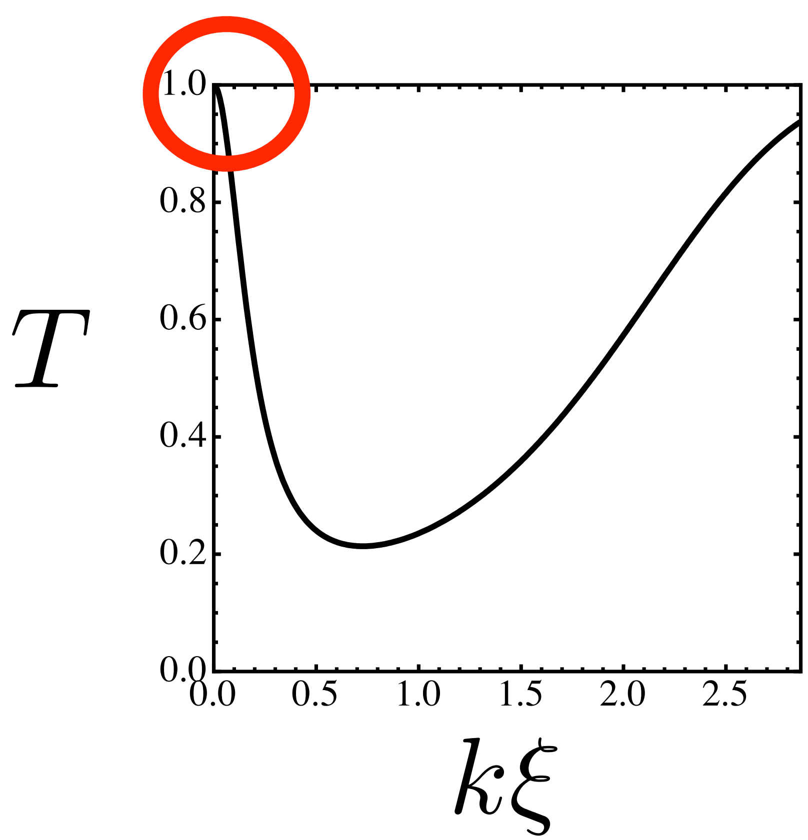
  $$E = \varepsilon$$

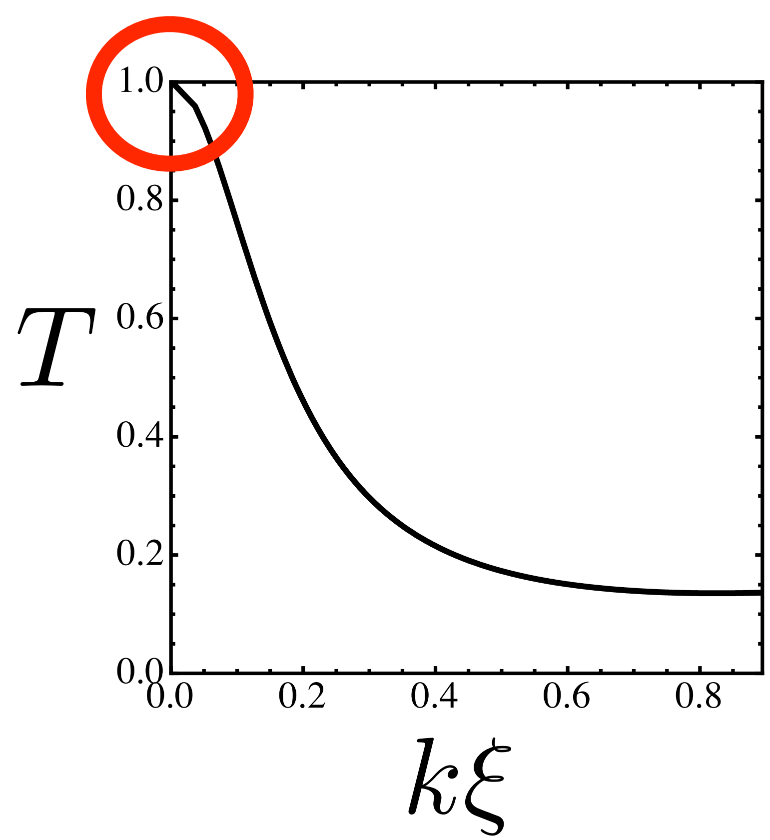
- quadratic spin fluctuation

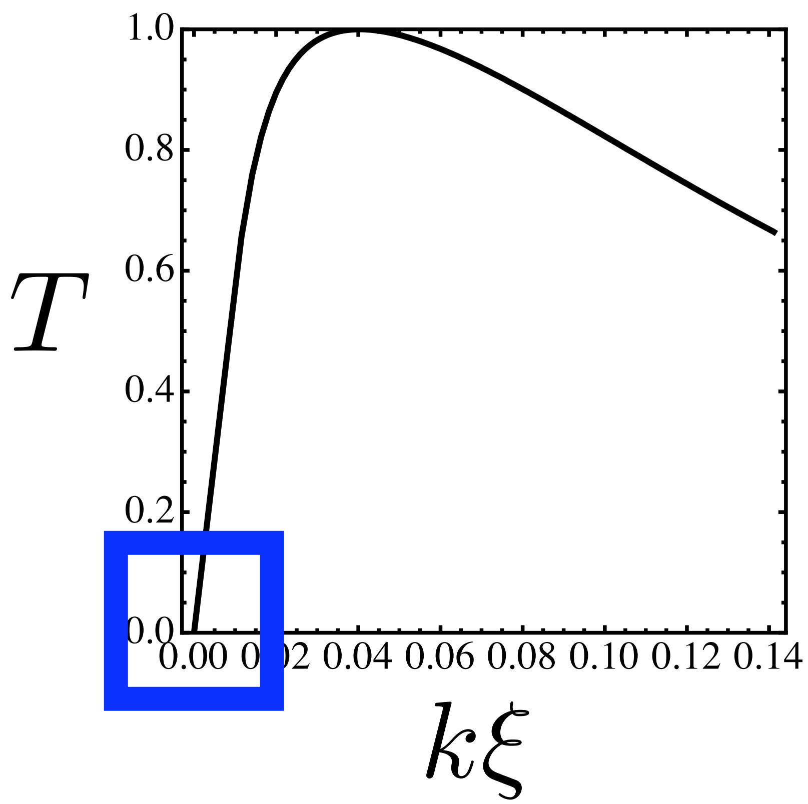
  m=+1 ⟶ m=-1

  $$E = \varepsilon + 2|c_2|n_0$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$





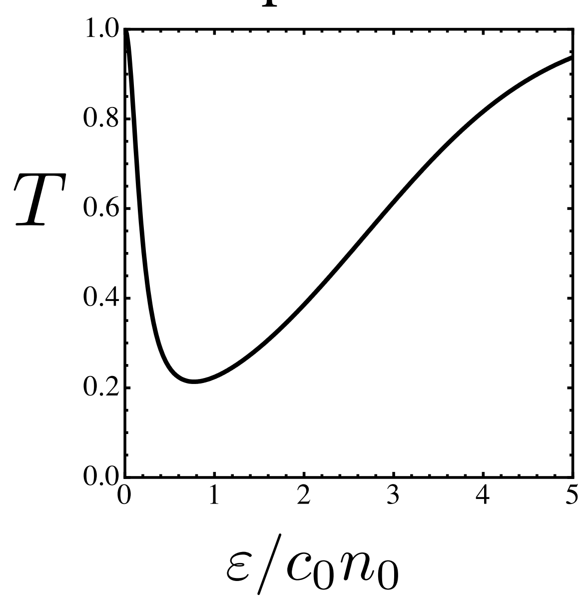


Results : Ferromagnetic state

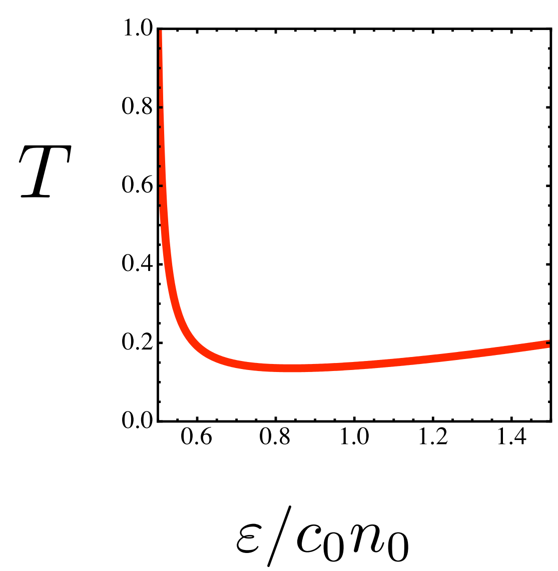
	$B = 0$	$B \neq 0$
fluctuation of order parameter Bogoliubov	○	○
spin fluctuation parabola	○	+ Zeeman shift ○
quadratic spin fluctuation parabola + energy gap	×	+ Zeeman shift ×

$B \neq 0$

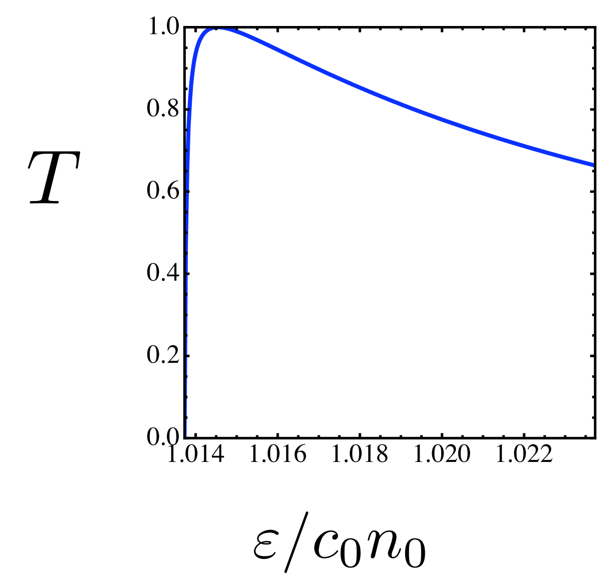
order parameter



spin



quadratic spin



Discussion

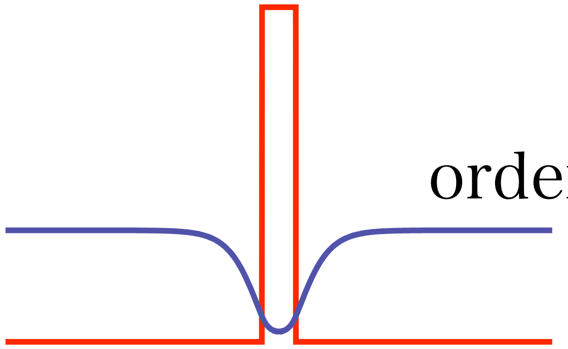
Q. Does perfect transmission depend on excitation spectra?

: Bogoliubov (phonon)?, parabolic?, with gap?

A. independent of these excitation spectra

Q. What is one of view to discuss the perfect transmission?

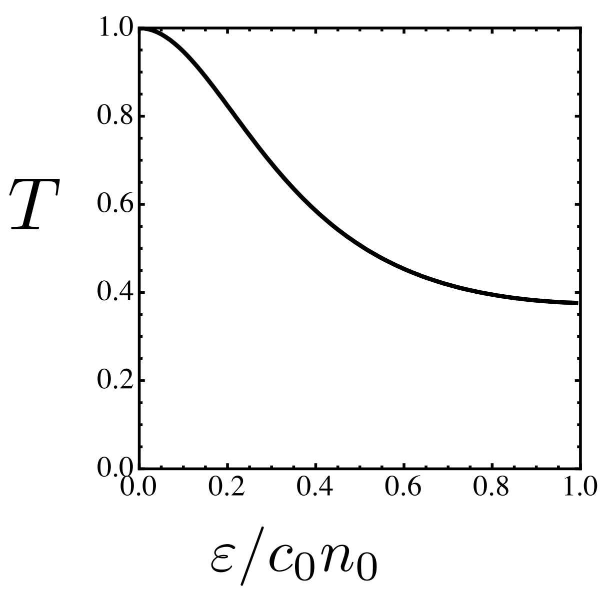
wave function of excited state in the long wavelength limit
= order parameter.



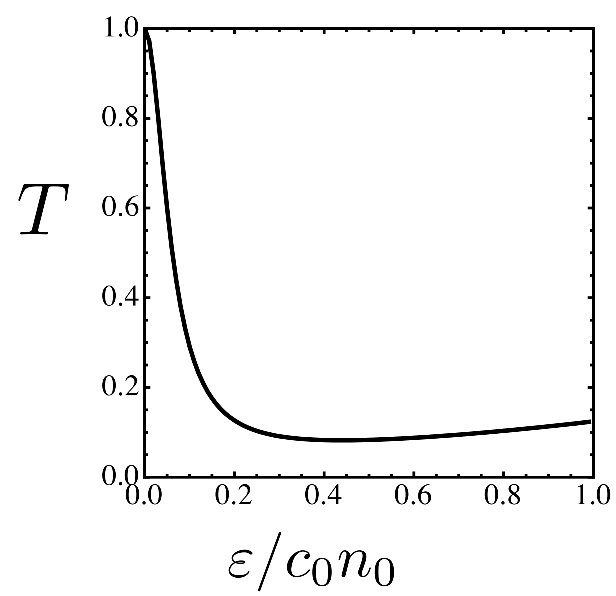
How about Polar State?

$B = 0$		$B \neq 0$ $c_0 = c_2$ integrable condition	
magnon δn Bogoliubov	○	m=+1 ⟶ m=+1 Bogoliubov	○
spin $\delta M_{\pm}$ Bogoliubov	○	m=-1 ⟶ m=- 1 Bogoliubov	○
		m=+1,-1 ⟶ m=0 Bogoliubov with gap	○

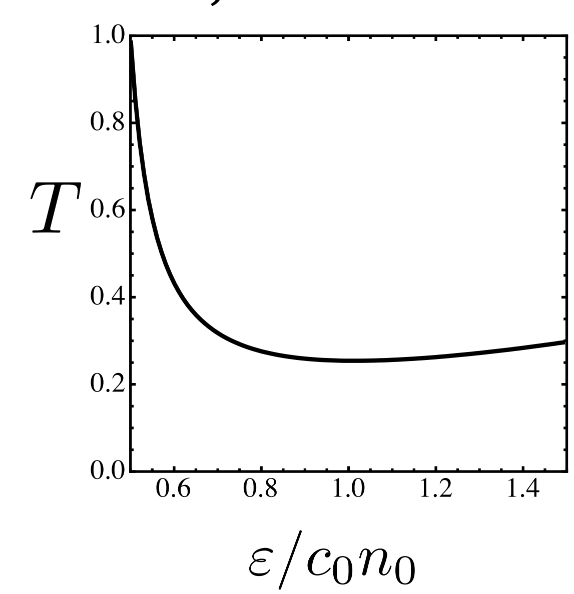
m=+1 ⟶ m=+1



m=-1 ⟶ m=- 1



m=+1,-1 ⟶ m=0



Summary and Outlook

Transmission coefficients of excitations in spin-1 BEC is discussed.

- Perfect transmission in the long wavelength limit is independent of its excitation spectra.
- Key is correspondence between the wavefunction of excited state and the order parameter.

- Polar state $c_0 \neq c_2$
- with spin current