Stability criterion of superfluidity in the presence of potential barrier

Yusuke Kato(A03)
（Dept. of Basic Science, Univ. Tokyo）

collaborators

S. Watabe （Dept. Phys. Univ.Tokyo)

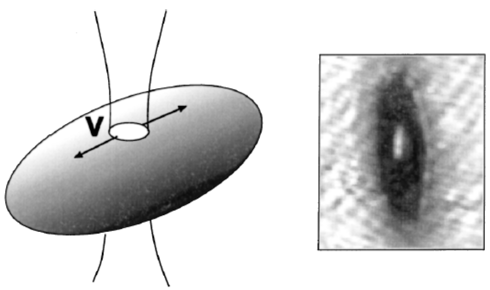
D. Takahashi（ Dept. of Basic. Science. Univ. Tokyo ）

Critical velocity of superfluidity （Earlier studies）

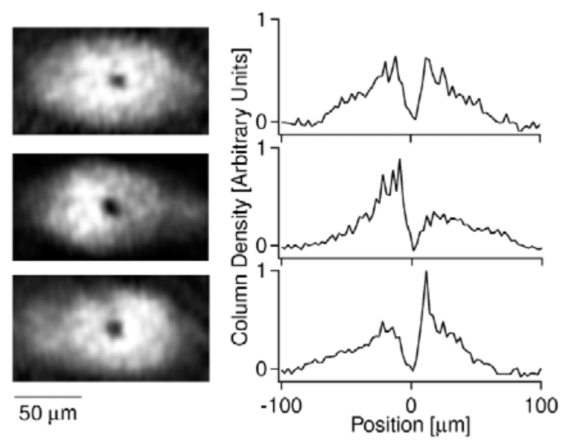
Landau's criterion $v_c = \min\left(\frac{\varepsilon(p)}{p}\right)$:= v_{cl}

In reality, v_c << $v_{c,\mathrm{Landau},}$; vortex creation in Cold Atoms

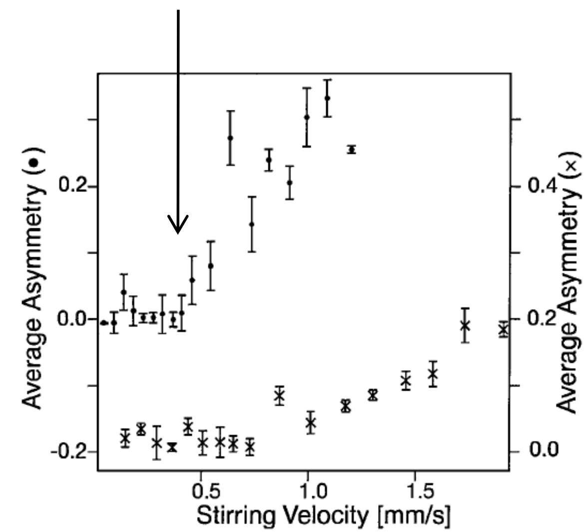
stirring condensate by laser beam



Raman et al. 1999



v_c



Onofrio et al. 2000

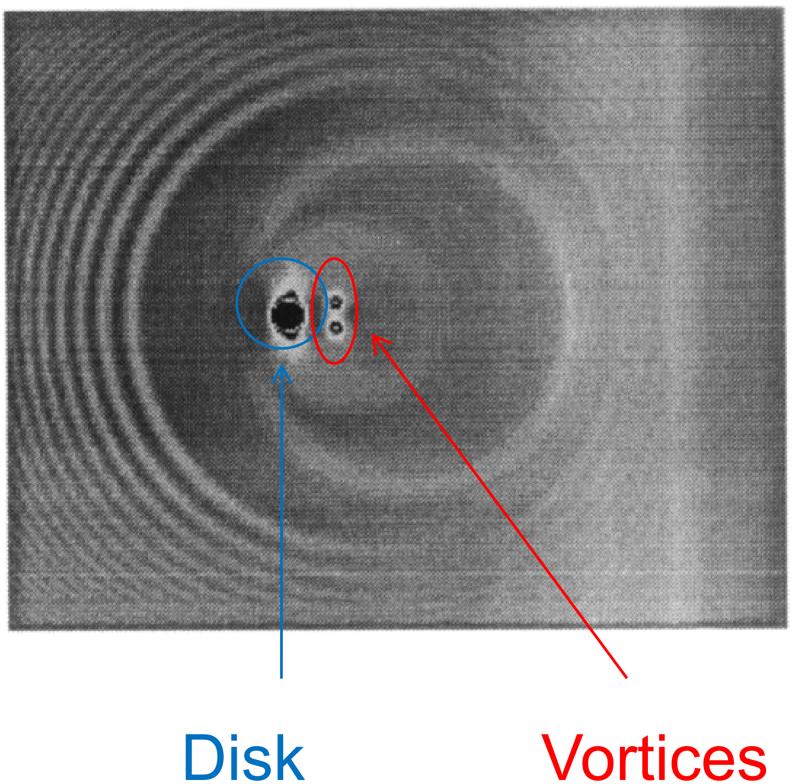
He4; Mobile impurity(Takahashi-Kono)

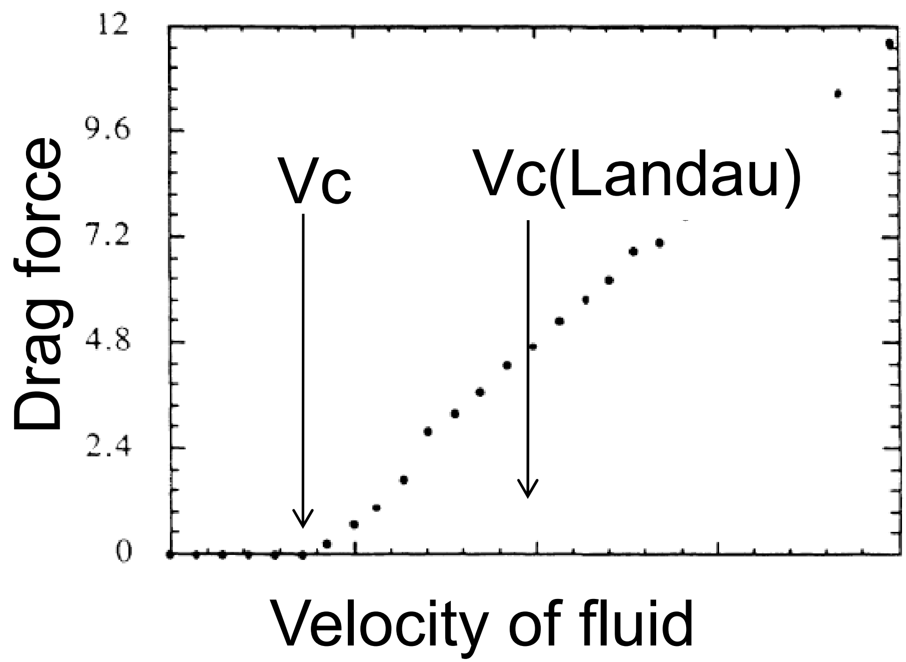
Vibrating wire (Yano et al.)

Numerical works of Gross-Pitaevskii eq.

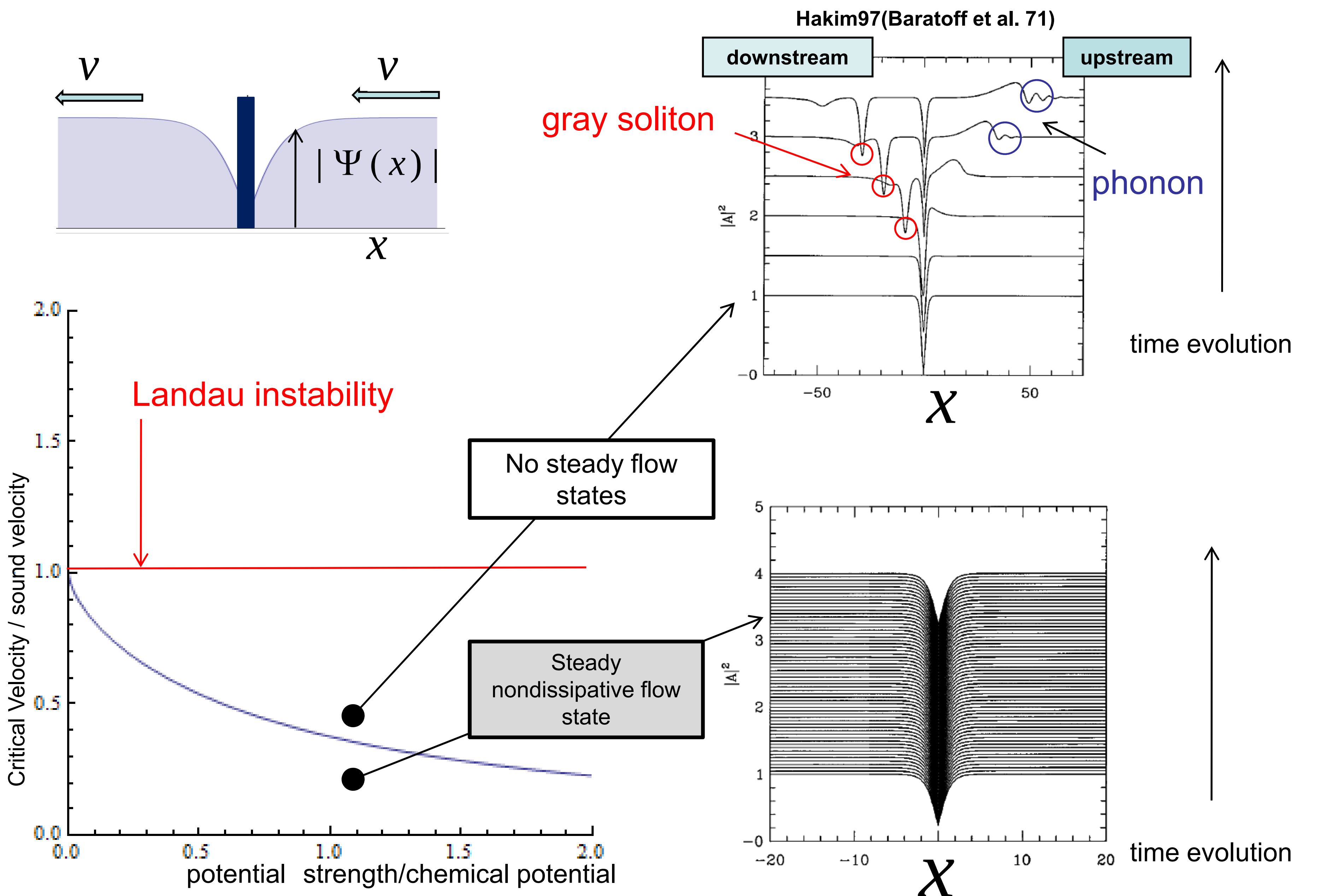
2D problem; Vc for flow around a disk

Frisch et al. 1992





Critical current of flow against the potential barrier（1dGP）



Summary of current status

- Flow in the presence of repulsive potential wall

  → <u>gray soliton (+phonon) creation</u>

- Flow in the presence of extremely small potential

  → <u>Landau instability</u>

  Flow around a disk

  → <u>vortex creation;</u>

Q. Does a unified criterion exist ?

Clue for solution

- Landau criterion →eigenenergy of excited states
- soliton generation →wavefunction of excited states

→ Spectral function

- Applicability to inhomogeneous system→spectral function of a local quantity

Our proposal: Description by local density spectral function

$$\hat{n}(x) := \hat{\psi}^\dagger(x)\hat{\psi}(x) - \langle \mathrm{g}|\hat{\psi}^\dagger(x)\hat{\psi}(x)|\mathrm{g}\rangle$$

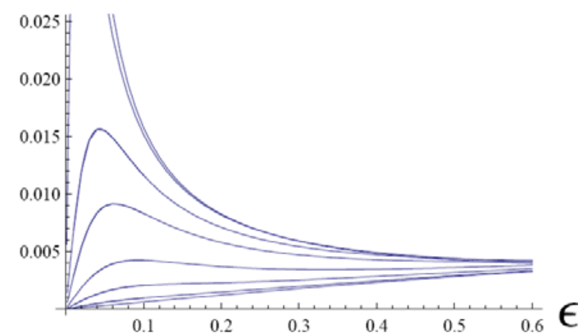
$$\rho(x,\epsilon) = \sum_l |\langle l|\hat{n}(x)|\mathrm{g}\rangle|^2 \delta(\epsilon - E_l + E_\mathrm{g})$$

When $\epsilon \longrightarrow 0 \quad \rho(\boldsymbol{r},\epsilon) \propto \epsilon^\beta$,

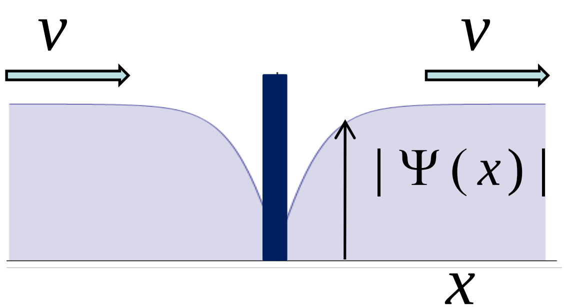
$$\beta = d, \qquad \text{for } v < v_\mathrm{c}$$
$$\beta < d, \qquad \text{for } v = v_\mathrm{c}$$

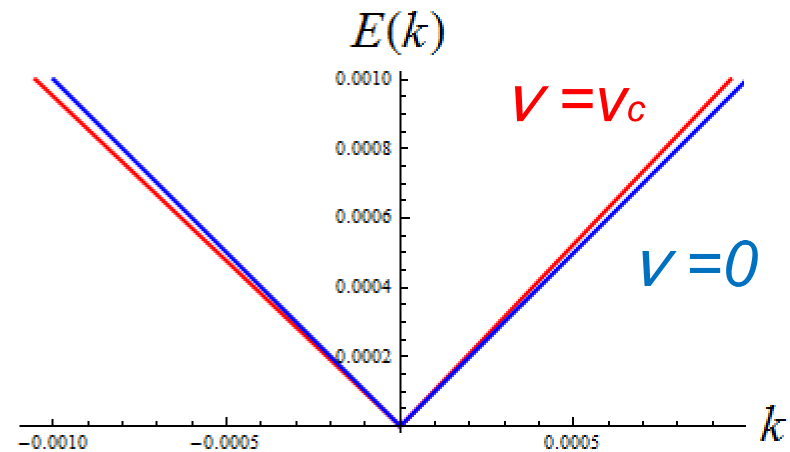
In *d*-dimension

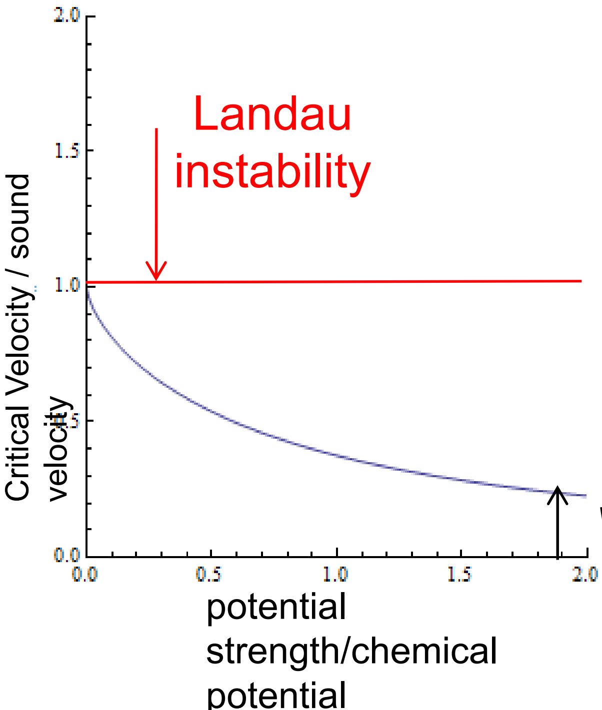
ρ(x=0,ϵ)

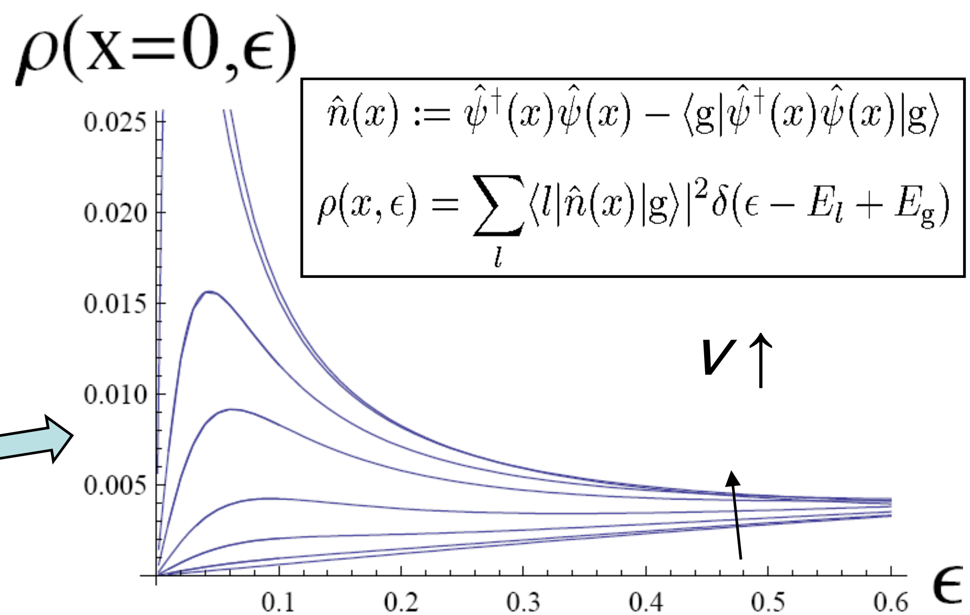


Flow in the presence of repulsive potential wall(1dGP+Bogoliubov)







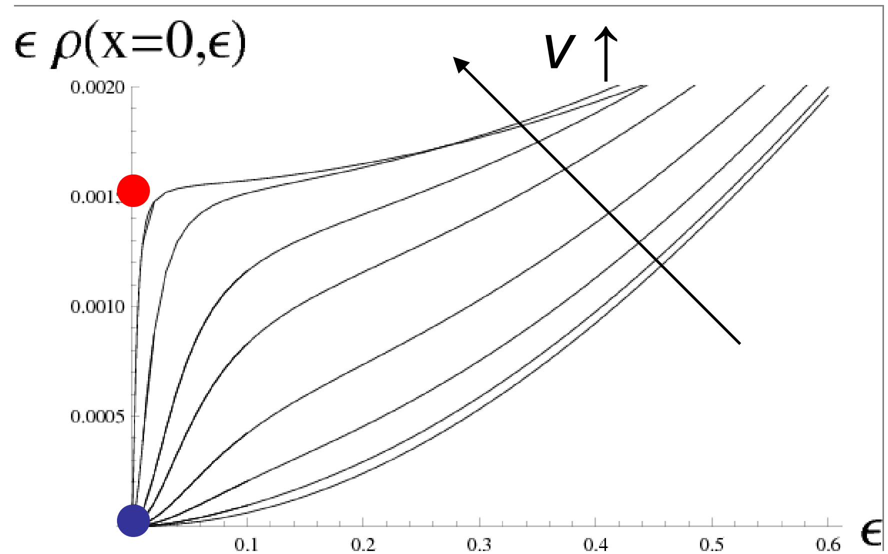


$$\hat{n}(x) := \hat{\psi}^{\dagger}(x)\hat{\psi}(x) - \langle \mathrm{g}|\hat{\psi}^{\dagger}(x)\hat{\psi}(x)|\mathrm{g}\rangle$$

$$\rho(x,\epsilon) = \sum_{l} |\langle l|\hat{n}(x)|\mathrm{g}\rangle|^{2}\delta(\epsilon - E_l + E_{\mathrm{g}})$$

Results for d=1
(repulsive potential)

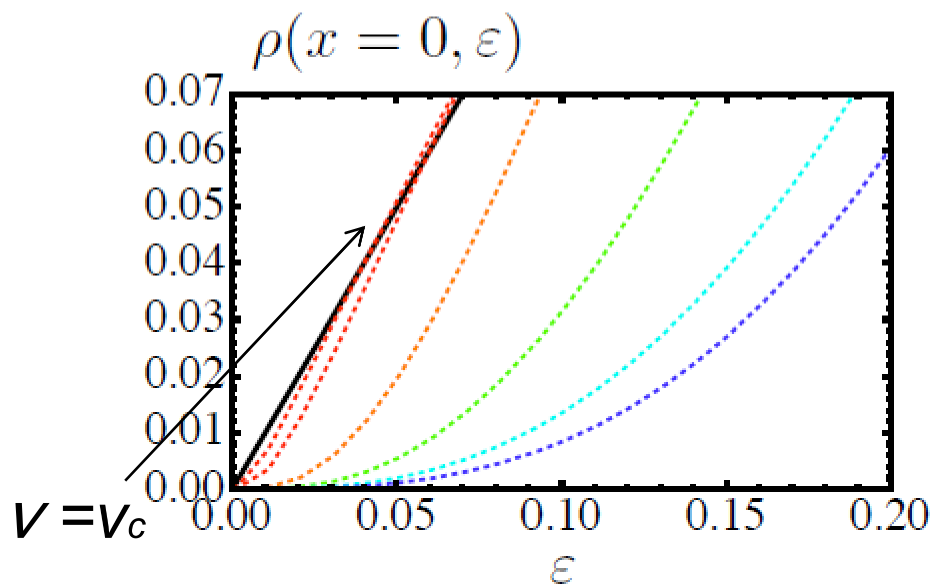
$$\rho(x,\epsilon) \propto 1/\epsilon \qquad v = v_c$$
$$\rho(x,\epsilon) \propto \epsilon \qquad v < v_c$$



For 1D system

Generalization to d=2,3
(repulsive potential)

$$\rho(x,\epsilon) \propto \epsilon^{d-2} \qquad v(x \to \pm\infty) = v_c$$
$$\rho(x,\epsilon) \propto \epsilon^{d} \qquad v(x \to \pm\infty) < v_c$$



For 3D system

Reduction to Landau's criterion (1/2)

In the weak barrier limit, system becomes spatially uniform and hence

$$\rho(x,\epsilon) = \rho(\epsilon) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^d} \underbrace{S(\boldsymbol{q},\epsilon)}$$

"Dynamical structure factor"

In the weak current limit of Bogoliubov theory,

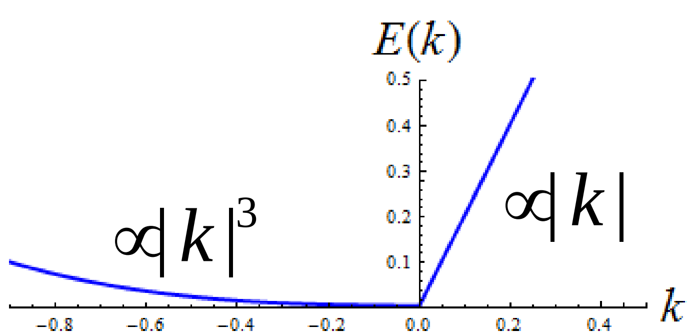
$$S(\boldsymbol{q},\epsilon) \propto |\boldsymbol{q}|\delta(\epsilon - E_{\mathrm{BG}}(\boldsymbol{q}))$$

$$\rho(\epsilon) \sim \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^d}|\boldsymbol{q}|\delta(\epsilon - E_{\mathrm{BG}}(\boldsymbol{q})) \quad \sim \epsilon \underbrace{\int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^d}\delta(\epsilon - E_{\mathrm{BG}}(\boldsymbol{q}))}$$

"Density of one-particle states"

$$\sim \epsilon \times \epsilon^{d-1} = \boxed{\epsilon^d}$$

On the critical current $v = v_{\mathrm{c,Landau}}$ of Bogoliubov theory,



$$\rho(\epsilon) \sim \boxed{\epsilon^{d-4/3}}$$

Reduction to Landau's criterion (2/2)

In the limit of weak barrier limit and v=0

Within the single mode approximation and using f-sum rule (Feynman) ,

$$E(\boldsymbol{q}) \sim \frac{\hbar^2|\boldsymbol{q}|^2}{2m \underbrace{\int \frac{\mathrm{d}\epsilon}{2\pi} S(\boldsymbol{q},\epsilon)}_{\text{Static structure factor}}}$$

Suppose that $E(\boldsymbol{q}) \propto |\boldsymbol{q}|^n$ holds, then

$S(\boldsymbol{q},\epsilon) \propto |\boldsymbol{q}|^{2-n}\delta(\epsilon - E(\boldsymbol{q}))$, from which

$\rho(\epsilon) \propto \epsilon^{\frac{2+d}{n}-2}$ follows.

When n>1, v_c(Landau)=0, which can be regarded as a result of

v(=0)=v_c when $\beta = \dfrac{2+d}{n} - 2 < d$ (⇔n>1)

Discussion

➢Long-time behavior of <u>local density autocorrelation function</u>

$$\langle n(\boldsymbol{r},t)n(\boldsymbol{r},0)\rangle \sim t^{-\alpha}, \quad t\to\infty$$

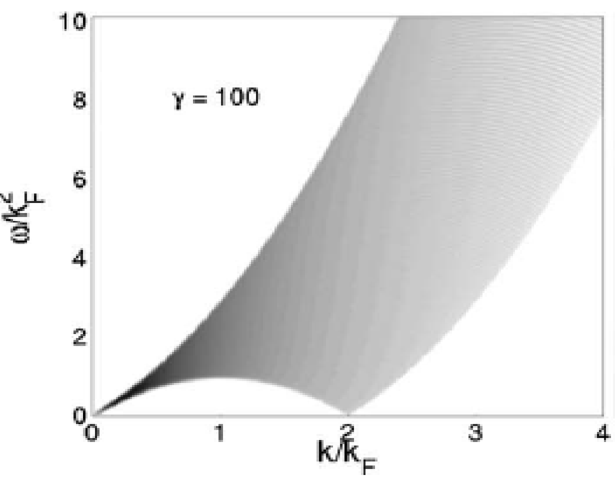
$$\epsilon\to 0 \quad \rho(\boldsymbol{r},\epsilon)\propto\epsilon^{\beta}\to\alpha=\beta+1$$

$\longrightarrow$

$$\begin{array}{ll}\alpha=d+1, & \text{for } v<v_c\\ \alpha<d+1, & \text{for } v=v_c\end{array}$$

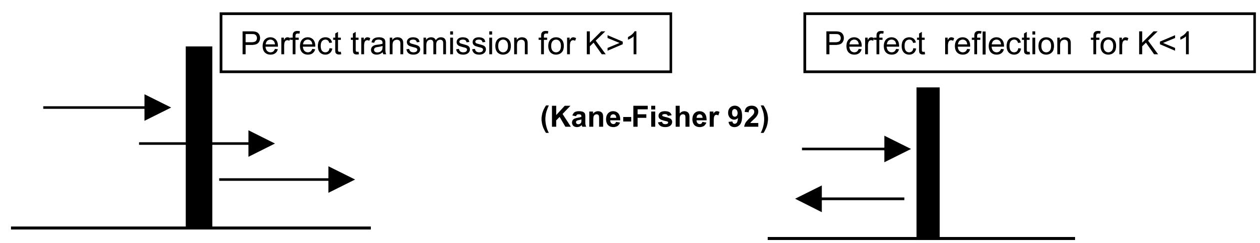
➢Applicability beyond mean field theory

➢Luttinger liquid (1d “superfluid”; quasi-long-range order) cf. Wada group(Nagoya)



Lieb-Leningner model(Caux 06)

Vc,Landau, =0, but….



K; Luttinger liquid parameter

$$\langle n(x,t)n(x,0)\rangle \sim \frac{A_0}{t^2}+\frac{A_1}{t^{2K}}+\frac{A_2}{t^{8K}}+\cdots, \quad t\to\infty$$

$$\alpha=2(=d+1), \qquad \text{for } K\geq 1 \quad \text{SF phase or Tonk-Girardeau}$$

$$\alpha=2K(<d+1), \qquad \text{for } K<1 \quad \text{CDW phase}$$

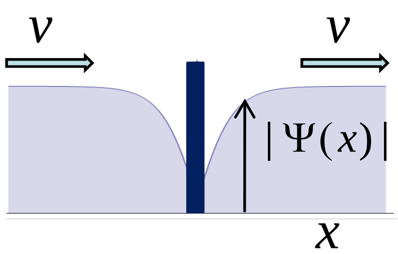
Conclusion

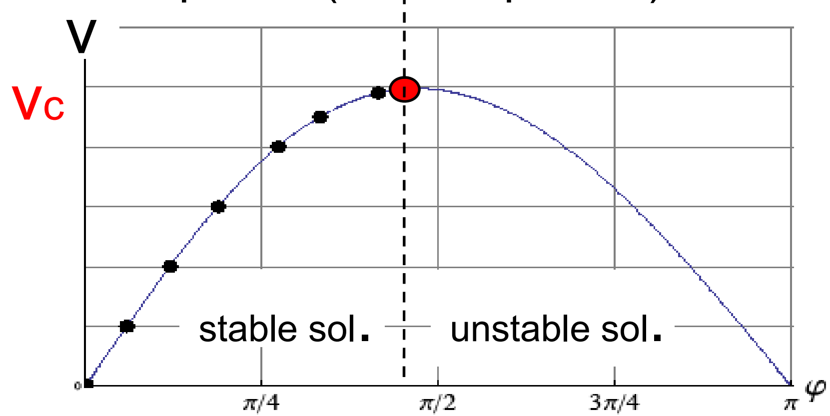
- A dynamical description of the criterion of instability of superfluid flow state is proposed

- It describes the instability of flow in the presence of potential barrier (due to soliton generation) and Landau instability in a unified way.

- Applicability “beyond mean-field-theory”
- Applicability to Luttinger-liquid is promising but it is to be further examined.
- Validity to instability of flow around a disk(cylinder) due to vortex generation is to be examined

Josephson (current-phase)relation





At v=vc, stable and unstable solutions are degenerate.

$$\Psi(x) = A(x)\exp[\mathrm{i}\Theta(x)]$$

Wavefunction of Bogoliubov excitations

$$S = u\mathrm{e}^{-\mathrm{i}\Theta} + v\mathrm{e}^{\mathrm{i}\Theta}, \quad G = u\mathrm{e}^{-\mathrm{i}\Theta} - v\mathrm{e}^{\mathrm{i}\Theta},$$

phase fluc. density fluc.

$$\begin{cases} |\psi|^2 = A^2\left[1 + \dfrac{2}{A}\mathrm{Re}(G\mathrm{e}^{-\mathrm{i}\epsilon t})\right] \\ \dfrac{\psi}{|\psi|} \simeq \mathrm{e}^{-\mathrm{i}\mu t + \mathrm{i}\Theta}\exp\left[\dfrac{\mathrm{i}}{A}\mathrm{Im}(S\mathrm{e}^{-\mathrm{i}\epsilon t})\right] \end{cases}$$

"Fetter's sol."

$$\begin{pmatrix} S \\ G \end{pmatrix} = \begin{pmatrix} A \\ 0 \end{pmatrix}$$

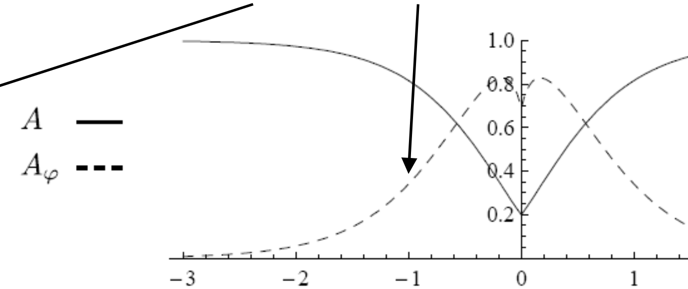
Exists for arbitrary velocity

"local density fluc. sol."

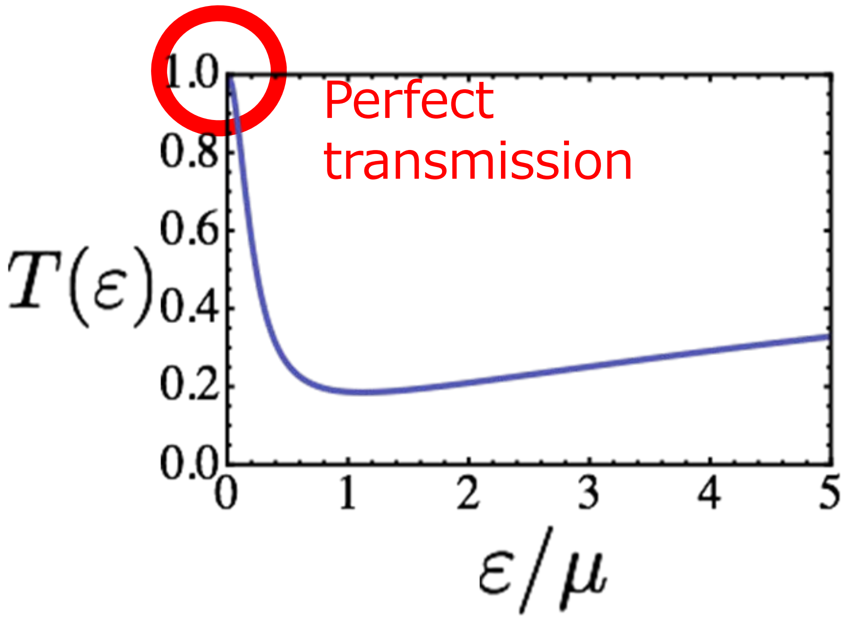
$$\begin{pmatrix} S \\ G \end{pmatrix} = \begin{pmatrix} -2\mathrm{i}qA\int_0^\infty \frac{1}{A^3}\frac{\partial A}{\partial \varphi} \\ \frac{\partial A}{\partial \varphi} \end{pmatrix}$$

Exists only for critical velocity velocity

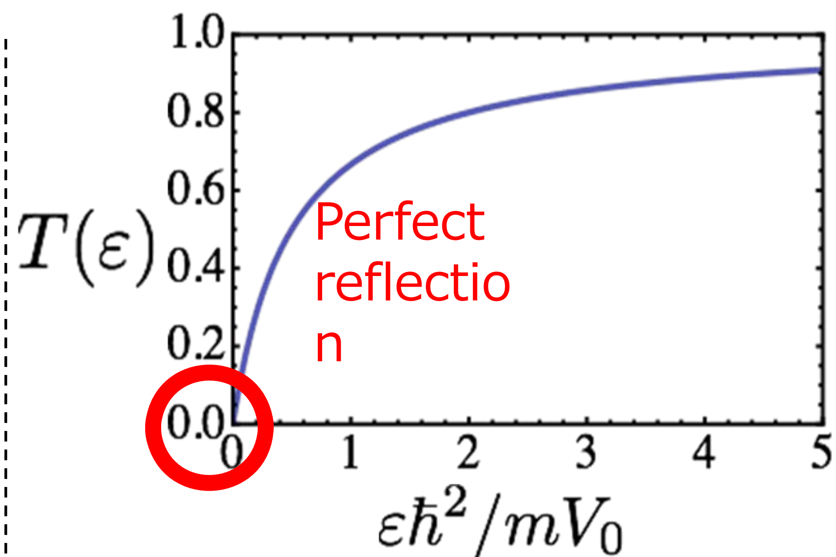
Localized density fluc.



"Anomalous Tunneling effect" = Perfect transmission in low energy limit

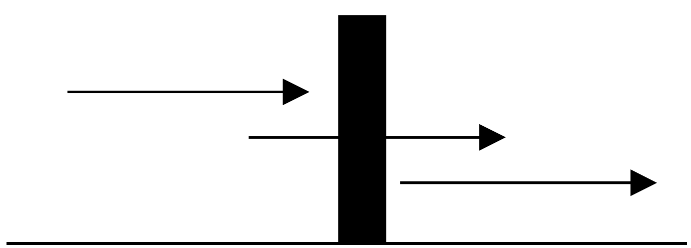


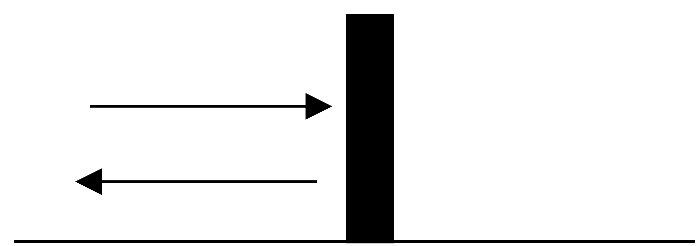
Bogoliubov exicitation $(V(x) = V_0\delta(x))$



Single particle in quantum mechanics

$(V(x) = V_0\delta(x))$





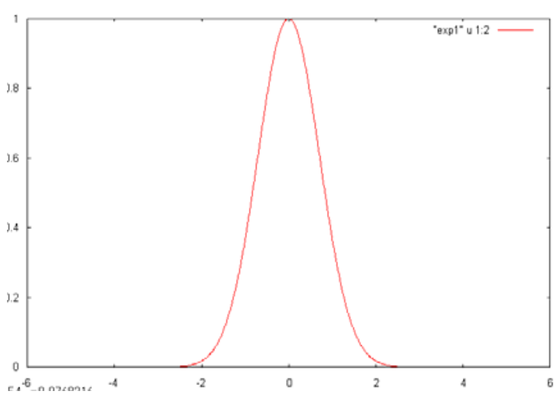
Gross-Pitaevskii eq. + Bogoliubov eq.

$$\{H_0(\mathbf{r}) + g|\Psi(\mathbf{r})|^2\}\Psi(\mathbf{r})=0 \qquad H_0(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{r}) - \mu$$

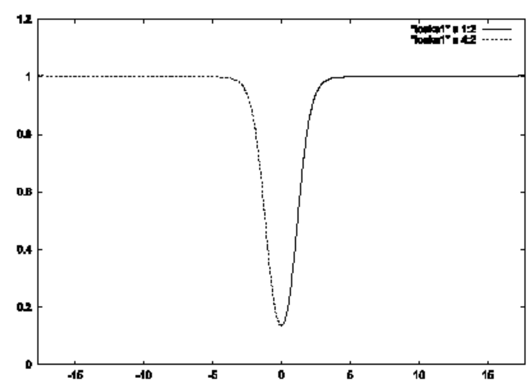
Gross-Pitaevskii equation (stationary)

$$\begin{pmatrix} H_0(\mathbf{r}) + 2g|\Psi(\mathbf{r})|^2 & -g[\Psi(\mathbf{r})]^2 \\ g[\Psi^*(\mathbf{r})]^2 & -H_0(\mathbf{r}) - 2g|\Psi(\mathbf{r})|^2 \end{pmatrix}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \varepsilon_i \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

Bogoliubov equation for $u_i(\mathbf{r}), v_i(\mathbf{r})$



Potential barrier (Gaussian)

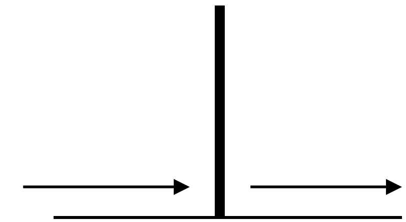


Condensate wavefunction

Earlier works

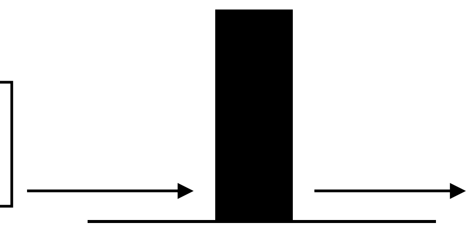
- Kovrizhin-Maksimov ‘01,Kovrizhin ’01

Finding perfect transmission in the low energy limit f Bogoliubov excitation in the presence of potential barrier **$V_0\delta(x)$**



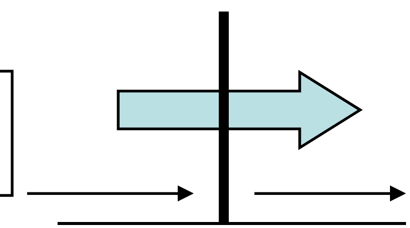
- Kagan et al. ’03 (“quasi resonance” scenario)

Discussion on the anamalous tunneling in the presence of **rectangular** potential.



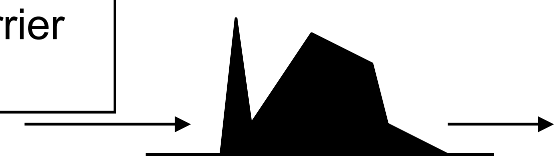
- Danshita et al. ‘06(“localized component” scenario)

Finding anomalous tunneling in the presence of potential barrier **$V_0\delta(x)$ and supercurrent**



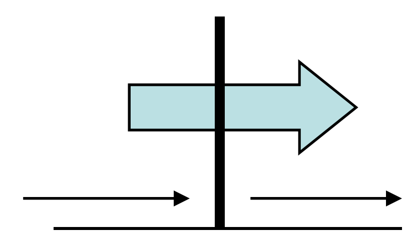
- Kato et al. ’08, Watabe-Kato ‘08 (“u,v→Ψ” scenario)

Finding anomalous tunneling in the presence of potential barrier with **arbitrary shape**



- Ohashi-Tsuchiya ’08 (“u,v→Ψ” scenario)

Detailed discussion on **Danshita’s exact solution** in the presence of potential barrier **$V_0\delta(x)$ and supercurrent**



“**Fetter’s solution**” (Fetter1972)

When $\varepsilon \to 0$ $\begin{pmatrix} u(\boldsymbol{r}) \\ v(\boldsymbol{r}) \end{pmatrix} = \begin{pmatrix} \Psi(\boldsymbol{r}) \\ \Psi^*(\boldsymbol{r}) \end{pmatrix}$ yields a solution of Bogoliubov eq.

“u,v→Ψ” scenario

$$\lim_{\epsilon \to 0} \begin{pmatrix} u \\ v \end{pmatrix} \propto \begin{pmatrix} \Psi \\ \Psi^* \end{pmatrix} \iff \text{Perfect transmission in low energy limit}$$

Low energy excitations inherit the transport properties from superfluidity of condensate.

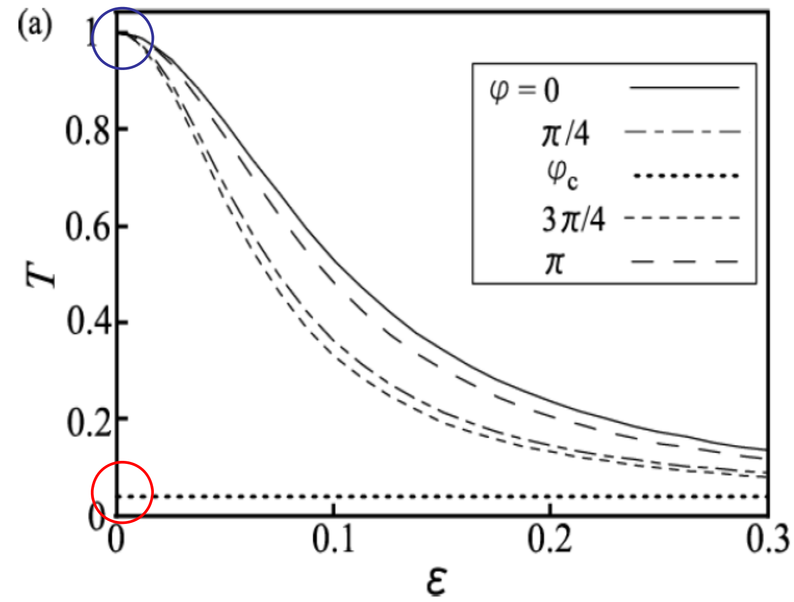
$$\{H_0(\mathbf{r}) + g|\Psi(\mathbf{r})|^2\}\Psi(\mathbf{r})=0 \qquad H_0(\mathbf{r}) = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V_{\rm ext}(\mathbf{r}) - \mu$$

Gross-Pitaevskii equation

$$\begin{pmatrix} H_0(\mathbf{r}) + 2g|\Psi(\mathbf{r})|^2 & -g[\Psi(\mathbf{r})]^2 \\ g[\Psi^*(\mathbf{r})]^2 & -H_0(\mathbf{r}) - 2g|\Psi(\mathbf{r})|^2 \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \varepsilon_i \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

Bogoliubov equation for $u_i(\mathbf{r}), v_i(\mathbf{r})$

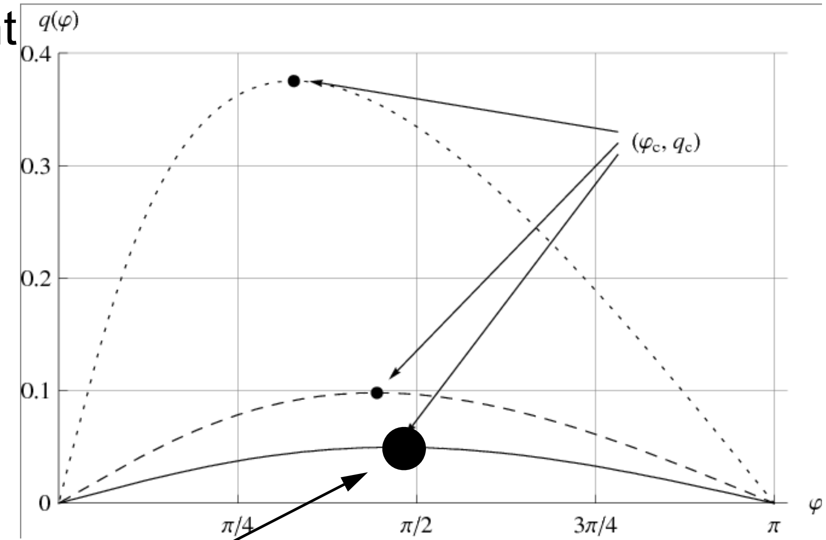
Transmission Prob. vs Energy



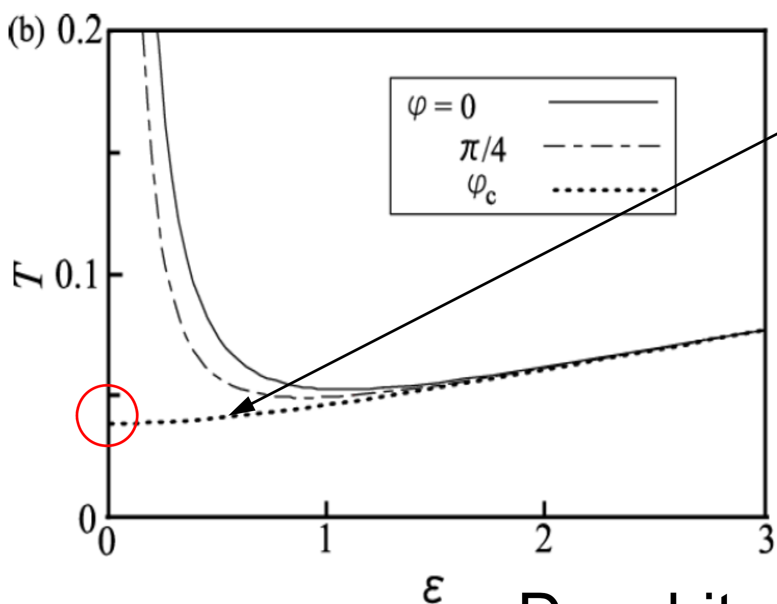
Josephson (current-phase)relation

Current

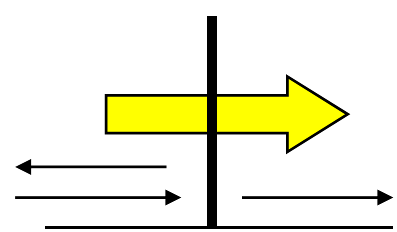
q



Phase



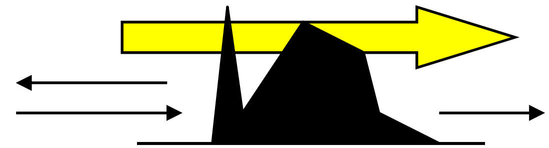
Partial transmission at critical current



Danshita et al 06

Tunneling problem of Bogoliubov mode **at critical current** in the presence of **generic form of potential barrier** (Takahashi-Kato JPSJ'09,No.2)

(1)Proof of partial transmission in the low energy limit



$$\epsilon \to 0, \quad T \to \frac{4q^2\eta^2}{(q^2+\eta^2)^2} (< 1 \text{ when } \eta \neq q)$$

$$\Psi(x) = A(x)\exp[\mathrm{i}\Theta(x)] \qquad \eta = -4q\int_0^\infty \frac{1}{A^3}\frac{\partial A}{\partial \varphi}$$

(2) Low energy limit of wave fn.of Excitation <u>do not coincide</u> with that of condensate.

$$\lim_{\epsilon\to 0}\begin{pmatrix} u \\ v \end{pmatrix} \propto \begin{pmatrix} \Psi \\ \Psi^* \end{pmatrix} - 2\mathrm{i}\,\frac{q-\eta}{q+\eta}\frac{\partial}{\partial\varphi}\begin{pmatrix} \Psi \\ \Psi^* \end{pmatrix}$$

"Fetter's sol." "local density fluc. sol."

(3) $S = u\mathrm{e}^{-\mathrm{i}\Theta} + v\mathrm{e}^{\mathrm{i}\Theta}, \quad G = u\mathrm{e}^{-\mathrm{i}\Theta} - v\mathrm{e}^{\mathrm{i}\Theta},$

phase fluc. density fluc.

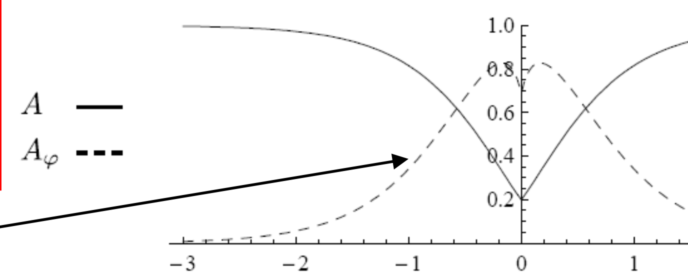
$$\begin{cases} |\psi|^2 = A^2\left[1 + \frac{2}{A}\mathrm{Re}(G\mathrm{e}^{-\mathrm{i}\epsilon t})\right] \\ \frac{\psi}{|\psi|} \simeq \mathrm{e}^{-\mathrm{i}\mu t + \mathrm{i}\Theta}\exp\left[\frac{\mathrm{i}}{A}\mathrm{Im}(S\mathrm{e}^{-\mathrm{i}\epsilon t})\right] \end{cases}$$

$$\begin{pmatrix} S \\ G \end{pmatrix} = \begin{pmatrix} A \\ 0 \end{pmatrix}$$

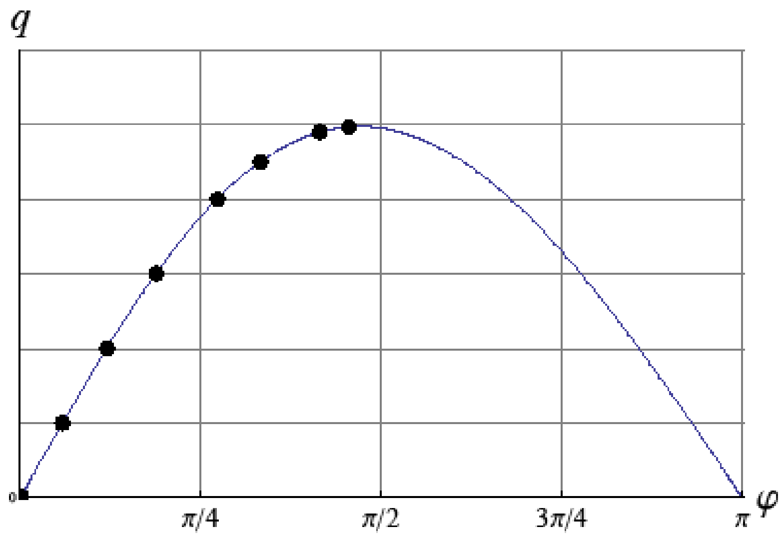
"phase fluctuation only

$$\begin{pmatrix} S \\ G \end{pmatrix} = \begin{pmatrix} -2\mathrm{i}qA\int_0^\infty \frac{1}{A^3}\frac{\partial A}{\partial\varphi} \\ \frac{\partial A}{\partial\varphi} \end{pmatrix}$$

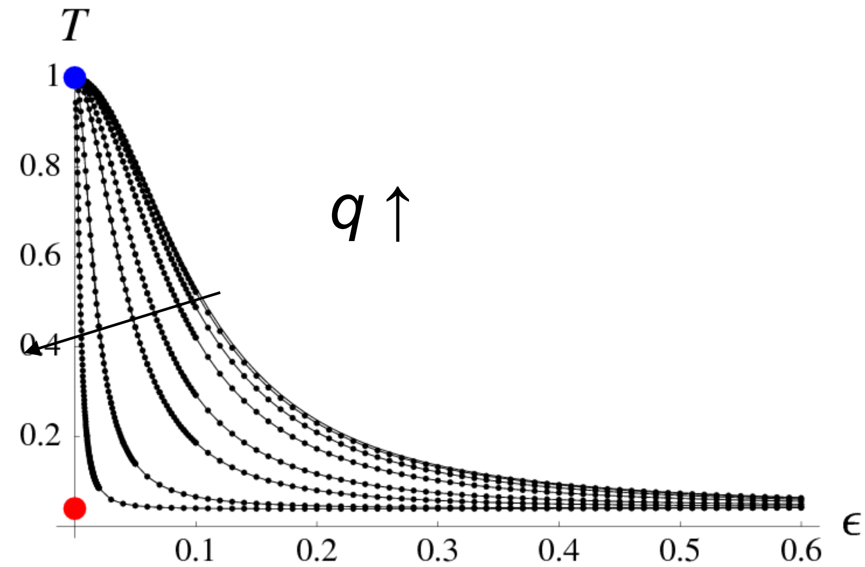
Localized density fluc.



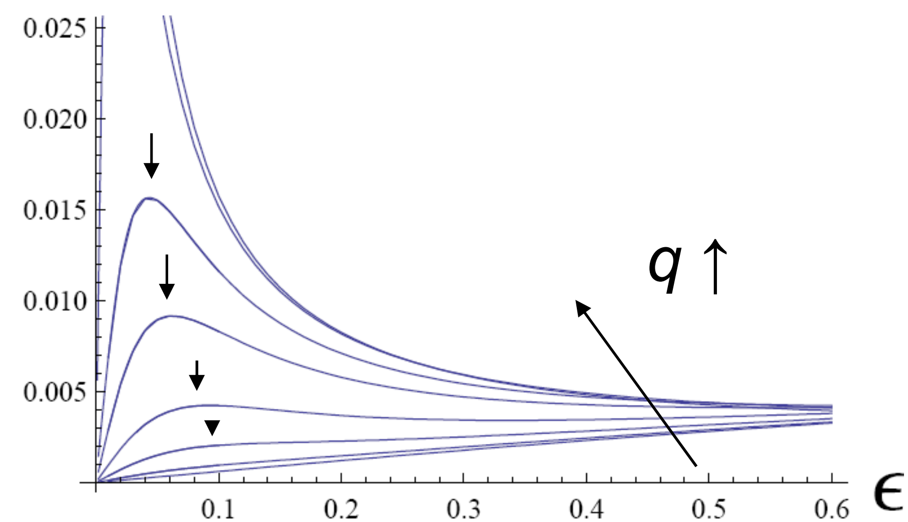
Josephson (current-phase)relation



Transmission Prob. vs Energy



$\rho(\mathrm{x}{=}0,\epsilon)$

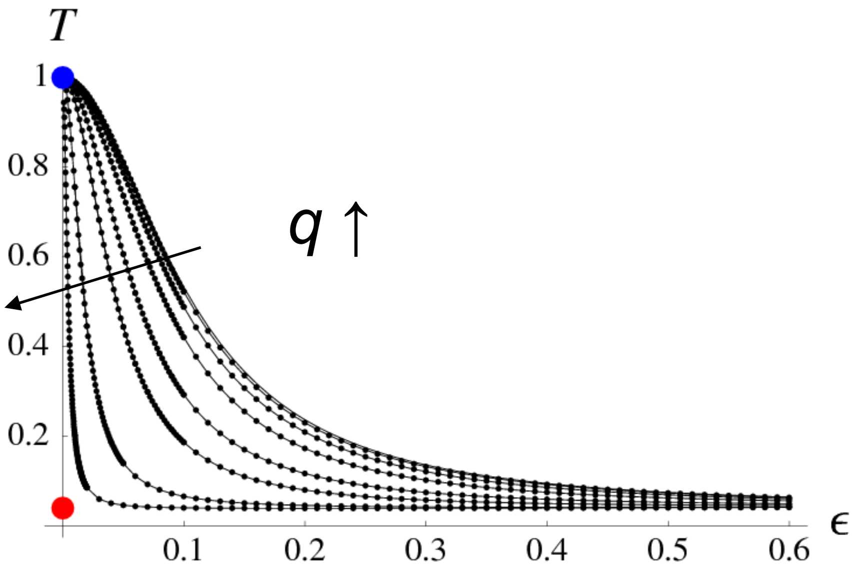


$$\hat{n}(x) := \hat{\psi}^\dagger(x)\hat{\psi}(x) - \langle \mathrm{g}|\hat{\psi}^\dagger(x)\hat{\psi}(x)|\mathrm{g}\rangle$$

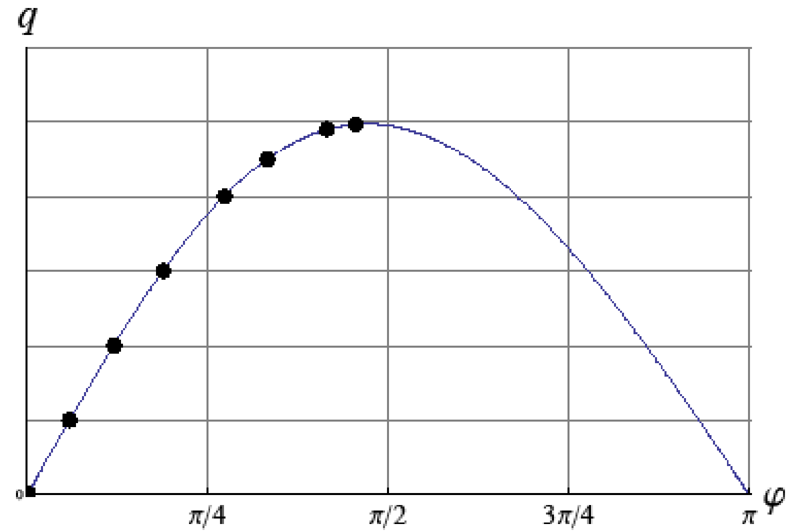
$$\rho(x,\epsilon) = \sum_l \langle l|\hat{n}(x)|\mathrm{g}\rangle|^2 \delta(\epsilon - E_l + E_{\mathrm{g}})$$

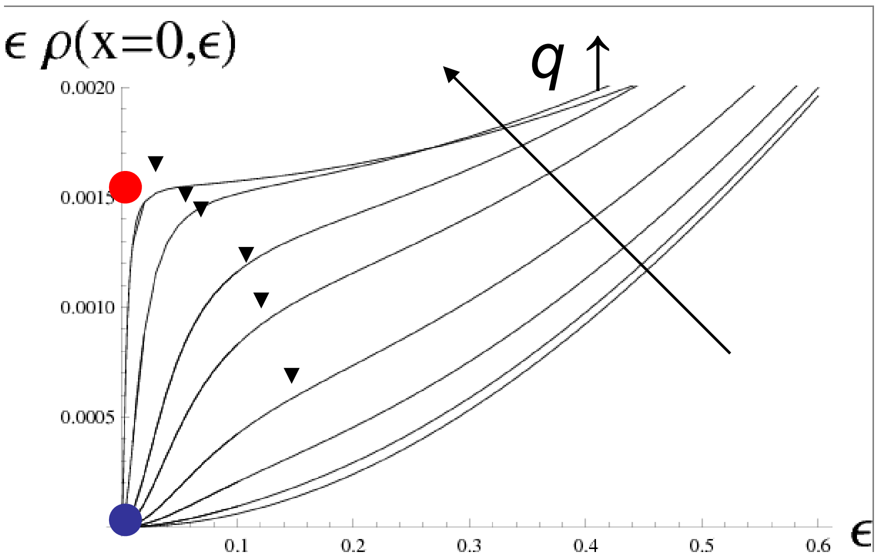
“Spectral function of local density fluctuation”

Transmission Prob. vs Energy



Josephson (current-phase)relation





- With supercurrent q↑, local density fluctuation increases.
- Cross over energy (▾) decreases, with q↑
- Quench of local density fluc. Leads to enhancement of transmission probability

Discussion; Importance of suppression of local density fluctuation

suppression of local density fluctuation

↓

Low energy excitation ; phase fluctuation

↓

$$\lim_{\epsilon \to 0} \begin{pmatrix} u \\ v \end{pmatrix} \propto \begin{pmatrix} \Psi \\ \Psi^* \end{pmatrix}$$

↓

Physical properties of low energy excitations are similar to those of condensate.

↓

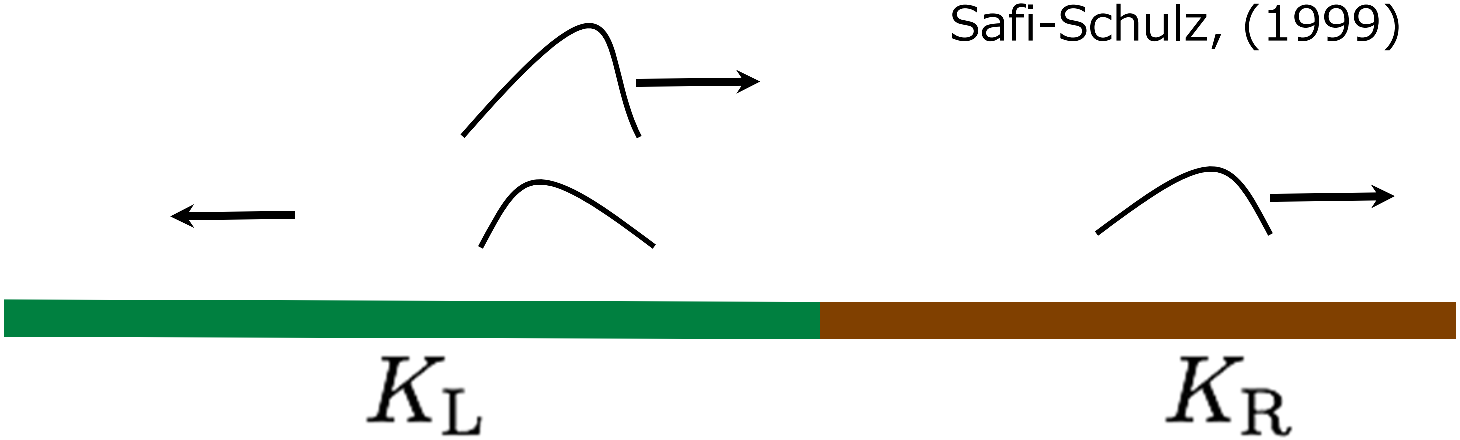
Perfect transmission in low energy limit

Related activities

- Anomalous Tunneling in Spinor BEC (Watabe); **P36**
- Linewidth (Takahashi); **P37**
- Relevance to physics of Tomonaga-Luttinger liquids(Watabe)
- Tunneling between BEC with different densities (Watabe)
- Finite Temperature (Nishiwaki)

Relevance to Tunneling of Luttinger liquid (Watabe-Kato)

Safi-Schulz, (1999)



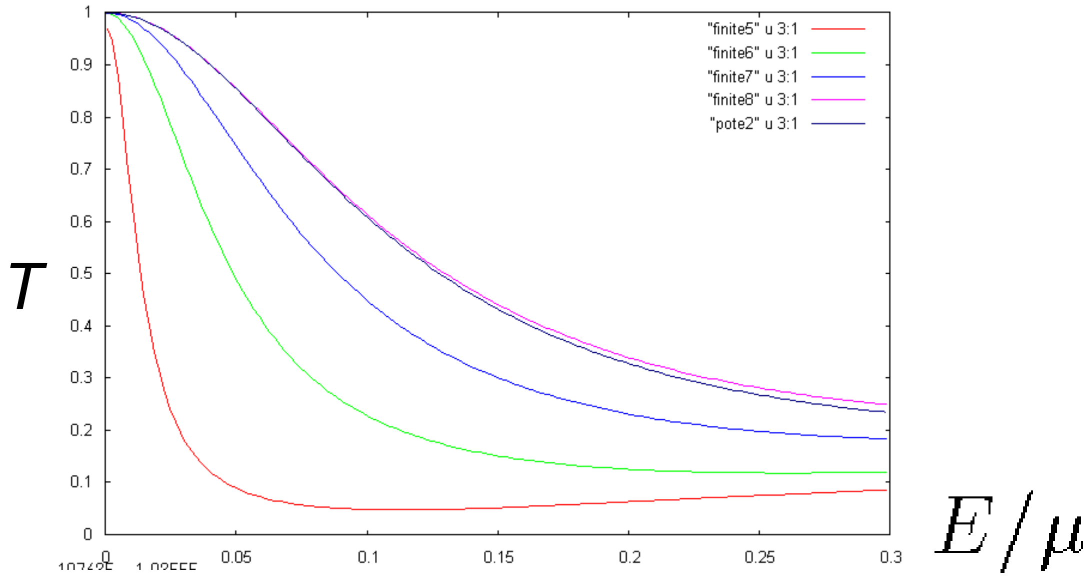
Luttinger parameter

$$T = \frac{4K_{\rm L}K_{\rm R}}{(K_{\rm L}+K_{\rm R})^2} \qquad R = \frac{(K_{\rm L}-K_{\rm R})^2}{(K_{\rm L}+K_{\rm R})^2}$$

Weakly interacting Bosons $K = \pi\hbar\sqrt{\frac{\rho_0}{mg}}$

$$T = \frac{4c_{\rm L}c_{\rm R}}{(c_{\rm L}+c_{\rm R})^2} \quad R = \frac{(c_{\rm L}-c_{\rm R})^2}{(c_{\rm L}+c_{\rm R})^2} \qquad c \equiv \sqrt{\frac{g\rho_0}{m}}$$

Finite Temperature, Popov approx.(Nishiwaki-Kato)



$$\underline{\{H_0(\mathbf{r}) + g|\Psi(\mathbf{r})|^2 + 2g\rho(\mathbf{r})\}\Psi(\mathbf{r}) = 0}$$ Stationary GP eq..

Eq. for excitation

$$(H_0(\mathbf{r}) + 2g|\Psi(\mathbf{r})|^2 + 2g\rho(\mathbf{r}))u_i(\mathbf{r}) - g\Psi(\mathbf{r})^2 v_i(\mathbf{r}) = \varepsilon_i u_i(\mathbf{r})$$
$$(H_0(\mathbf{r}) + 2g|\Psi(\mathbf{r})|^2 + 2g\rho(\mathbf{r}))v_i(\mathbf{r}) - g\Psi^*(\mathbf{r})^2 u_i(\mathbf{r}) = -\varepsilon_i v_i(\mathbf{r})$$

$$\rho(\mathbf{r}) = \sum_i \left[|u_i(\mathbf{r})|^2 f^{\mathrm{B}}(\varepsilon_i) + |v_i(\mathbf{r})|^2 (f^{\mathrm{B}}(\varepsilon_i) + 1)\right]$$ f^{B}; Bose fn.

Future work

- <u>Instability of current carrying Luttinger liquid in the presence of impurity</u>

- <u>Local density fluctuation spectrum and instability of superflow around an obstacle</u>

Critical current of flow against the potential barrier（1dGP）