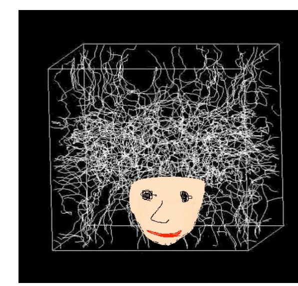
Magnetic Resonance in Cold Bose Gases

Masashi Yasunaga



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Contents

- concepts of our study
- Rabi-Josephson oscillation
- summary

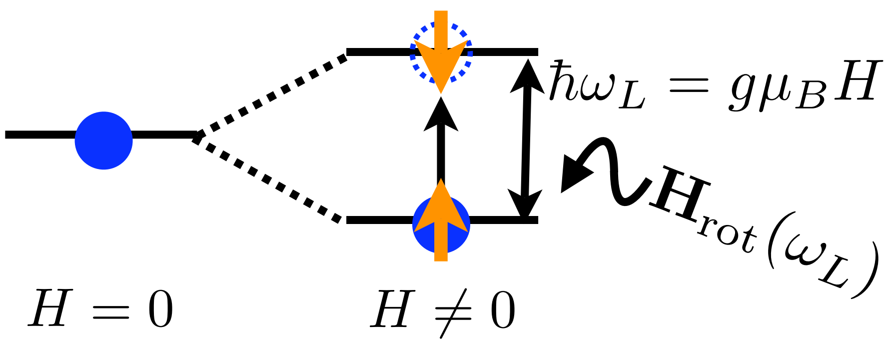
Concept of our study

Our purpose

Magnetic resonance

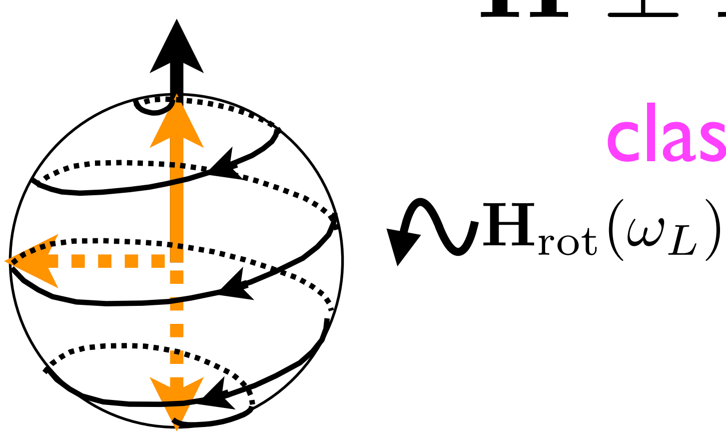
$$\mathbf{H} = H\hat{z}$$

quantum



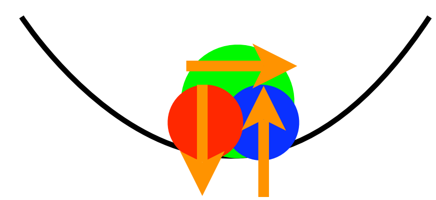
$$\mathbf{H} \perp \mathbf{H}_{rot}$$

classical



+

Spinor Bose-Einstein condensates



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system of a spin-1 BEC[1,2]

$$\mathcal{H} = \int d\mathbf{r} \left[\frac{\hbar^2}{2M} \nabla \hat{\psi}^\dagger_\alpha \cdot \nabla \hat{\psi}_\alpha + U \hat{\psi}^\dagger_\alpha \hat{\psi}_\alpha + \frac{c_0}{2} \hat{\psi}^\dagger_\alpha \hat{\psi}^\dagger_\beta \hat{\psi}_\beta \hat{\psi}_\alpha + \frac{c_2}{2} \hat{\psi}^\dagger_\alpha \hat{\psi}^\dagger_\beta \mathbf{F}_{\alpha\delta} \cdot \mathbf{F}_{\beta\gamma} \hat{\psi}_\gamma \hat{\psi}_\delta \right]$$

$$\alpha, \beta = 0, \pm 1$$

interaction parameters

$$c_0 = (g_0 + 2g_2)/3$$
$$c_2 = (g_2 - g_0)/3$$
$$g_i = 4\pi\hbar^2 a_i/M$$

s-wave scattering length a_i

ground state

$c_2 < 0$ $^{87}\mathrm{Rb}$ **ferromagnetic**

$c_2 > 0$ $^{23}\mathrm{Na}$ **polar or antiferro.**

[1] Tin-Lun Ho Phys. Rev. Lett. **81**, 742 (1998).
[2] T. Ohmi and K. Machida J. Phys. Soc. Jpn. **67**, 1822 (1998).

The Hamiltonian for MR

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_Z + \mathcal{H}_s + \mathcal{H}_{dd}$$

$$\mathcal{H}_0 = \int d\mathbf{r} \left[\frac{\hbar^2}{2M} \nabla \hat{\psi}^\dagger_\alpha \cdot \nabla \hat{\psi}_\alpha + U \hat{\psi}^\dagger_\alpha \hat{\psi}_\alpha + \frac{c_0}{2} \hat{\psi}^\dagger_\alpha \hat{\psi}^\dagger_\beta \hat{\psi}_\beta \hat{\psi}_\alpha \right]$$

$$\mathcal{H}_s = \int d\mathbf{r} \frac{c_2}{2} \hat{\psi}^\dagger_\alpha \hat{\psi}^\dagger_\beta \mathbf{F}_{\alpha\delta} \cdot \mathbf{F}_{\beta\gamma} \hat{\psi}_\gamma \hat{\psi}_\delta$$

Zeeman effect

$$\mathcal{H}_Z = \int d\mathbf{r} \hat{\psi}^\dagger_\alpha \langle \alpha | \hbar \gamma \mathbf{F} \cdot \mathbf{H} | \beta \rangle \hat{\psi}_\beta$$

Magnetic dipole-dipole interaction

$$\mathcal{H}_{dd} = \frac{c_{dd}}{2} \int d\mathbf{r} \int d\mathbf{r}' \hat{\psi}^\dagger_\alpha \hat{\psi}^\dagger_\beta \frac{\mathbf{F}_{\alpha\delta} \cdot \mathbf{F}_{\beta\gamma} - 3(\mathbf{F}_{\alpha\delta} \cdot \mathbf{e})(\mathbf{F}_{\beta\gamma} \cdot \mathbf{e})}{|\mathbf{r} - \mathbf{r}'|^3} \hat{\psi}_\gamma \hat{\psi}_\delta$$

Magnetic resonance in the Bose gas

E_Z / E_s

I

paramagnetic resonance

Rabi Oscillation

+ Josephson effects

+ phase separation

nonlinearity

Spin Echo[3]

(anti-)ferromagnetic resonance

$E_Z \sim E_{dd}$

[3] M. Yasunaga and M. Tsubota Phys. Rev. Lett. **101**, 0440201 (2008).

The spin-1 BECs

To compare general MR with the condensate system we transform GP eq. with *single spatial mode approximation*(SMA)[4].

$$i\hbar\frac{\partial\psi_\alpha}{\partial t} = \left(-\frac{\hbar^2}{2M}\nabla^2 + U - \mu\right)\psi_\alpha - \hbar\gamma H_i F^i_{\alpha\beta}\psi_\beta + c_0\psi^*_\beta\psi_\beta\psi_\alpha + c_2 F_i F^i_{\alpha\beta}\psi_\beta$$

$$\psi_i(\mathbf{r},t) = \sqrt{N}\xi_i(t)\phi(\mathbf{r})\exp(-i\mu t/\hbar)$$

SMA

$$\left(-\frac{\hbar}{2m}\nabla^2 + V - n\right)\phi = \mu\phi$$

$$c_0 \gg |c_2|$$

$$i\hbar\frac{d}{dt}|\xi\rangle = (\hat{H}_z + \hat{H}_{int})|\xi\rangle$$

spinor $|\xi\rangle = (\xi_1, \xi_0, \xi_{-1})^T$

density matrix $\rho = |\xi\rangle\langle\xi|$

Zeeman term $\hat{H}_z = -\hbar\gamma\mathbf{F}\cdot\mathbf{H}$

nonlinear interaction term

$$\begin{aligned}\hat{H}_{int} &= c_2\{E - \sum_i (1+|i|)\langle -i|\rho(t)|-i\rangle|i\rangle\langle i| \\ &+ \sum_{|i-j|=1}\langle -j|\rho(t)|-i\rangle|i\rangle\langle j|\}\end{aligned}$$

$$\begin{aligned}|1\rangle &= (1,0,0)^T \\ |0\rangle &= (0,1,0)^T \\ |-1\rangle &= (0,0,1)^T\end{aligned}$$

[4] H. Pu *et al.*, Phys. Rev. A **60**, 1463 (1999).

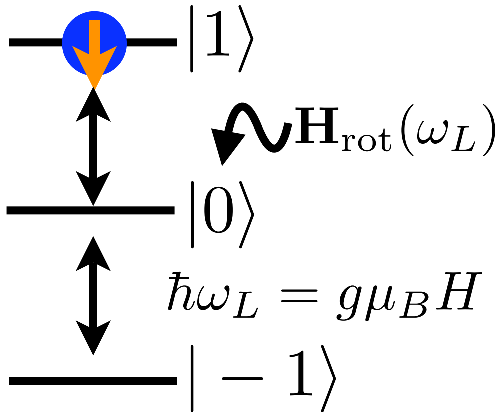
Rabi Oscillation for spin-1

In the limit $c_2/(g\mu_B H) \sim 0$

$$i\hbar\frac{d}{dt}|\xi\rangle = \hat{H}_z(t)|\xi\rangle$$

$$\hat{H}_z = -\hbar\gamma\mathbf{F}\cdot(H\hat{z} + \mathbf{H}_{rot})$$

$$\mathbf{H}_{rot} = H_1\cos\omega t\hat{x} + H_1\sin\omega t\hat{y}$$



$$|\xi\rangle = e^{i\omega\hat{F}_z t}|\eta\rangle$$

$$i\hbar\frac{d}{dt}|\eta\rangle = \hat{H}'_z|\eta\rangle \qquad \hat{H}'_z = \hbar\Delta\hat{n}\cdot\mathbf{F}$$

$$\Delta = \sqrt{\sigma^2 + \omega_1^2}$$

$$\hat{n} = \left(\frac{\omega_1}{\Delta}, 0, \frac{\sigma}{\Delta}\right)$$

$$\sigma = \omega - \omega_L$$

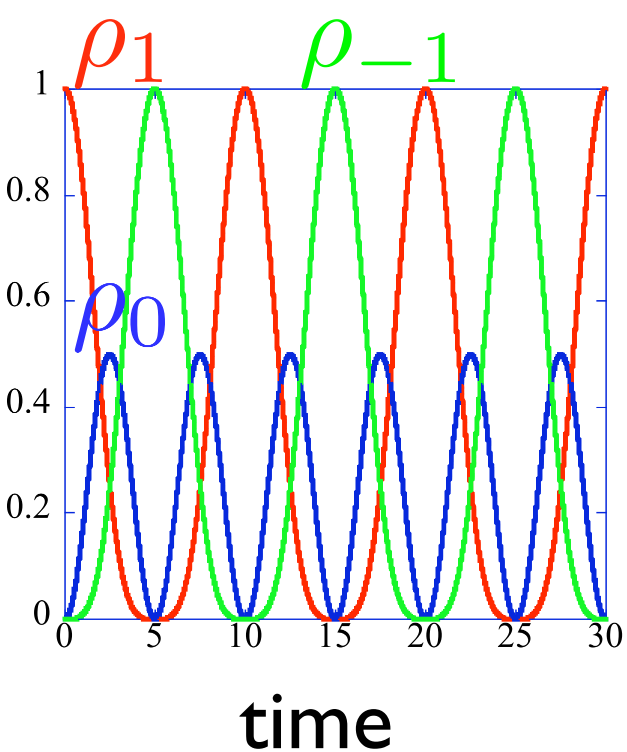
$$\omega = \omega_1$$

$$\rho_{11} = |\langle 1|\xi\rangle|^2 = \cos^4(\omega_1 t/2)$$

$$\rho_{00} = |\langle 0|\xi\rangle|^2 = \sin^2(\omega_1 t/2)$$

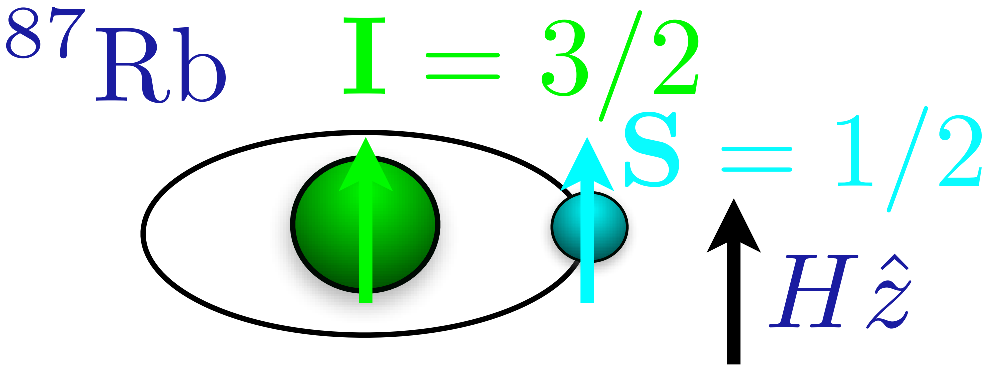
$$\rho_{-1-1} = |\langle -1|\xi\rangle|^2 = \sin^4(\omega_1 t/2)$$

$$\omega_1 = \gamma H_1$$



quadratic Zeeman effect

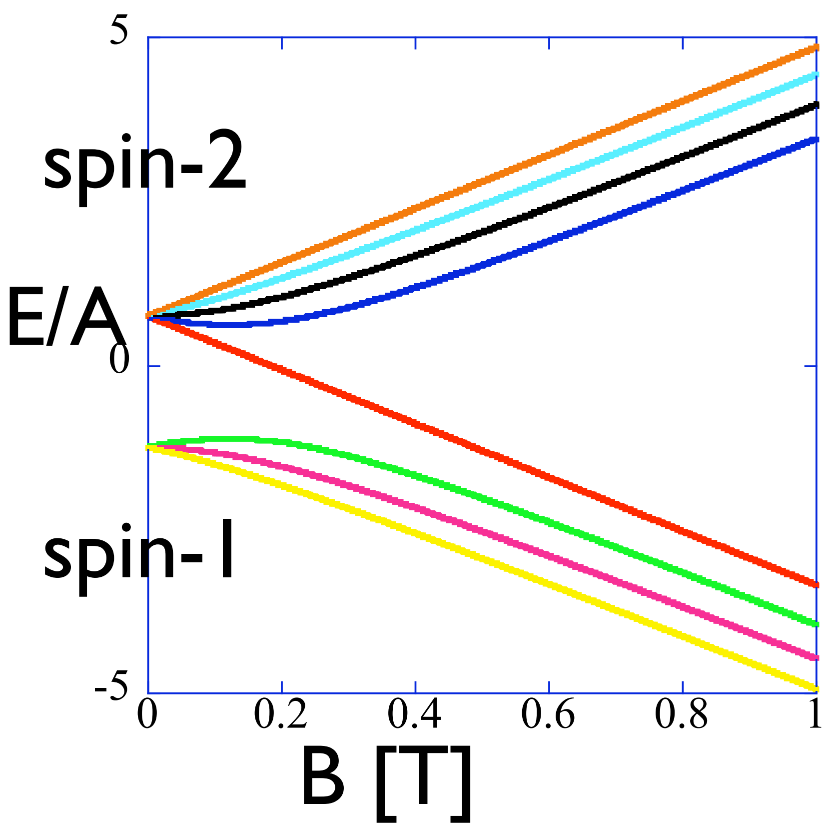
hyperfine structure



Breit-Rabi Hamiltonian

$$\hat{H} = A\mathbf{I}\cdot\mathbf{S} + CS_z + DI_z$$

$$A = \Delta E_{hfs}/2$$
$$C = g\mu_B B$$
$$D = g_N \mu_N B$$



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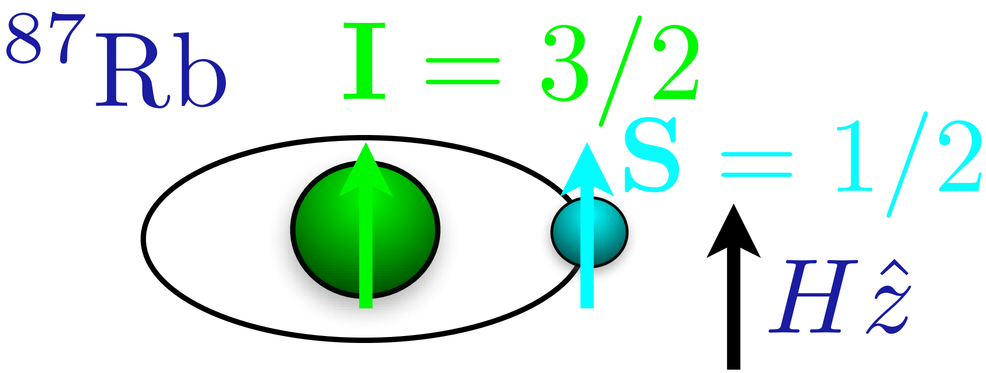

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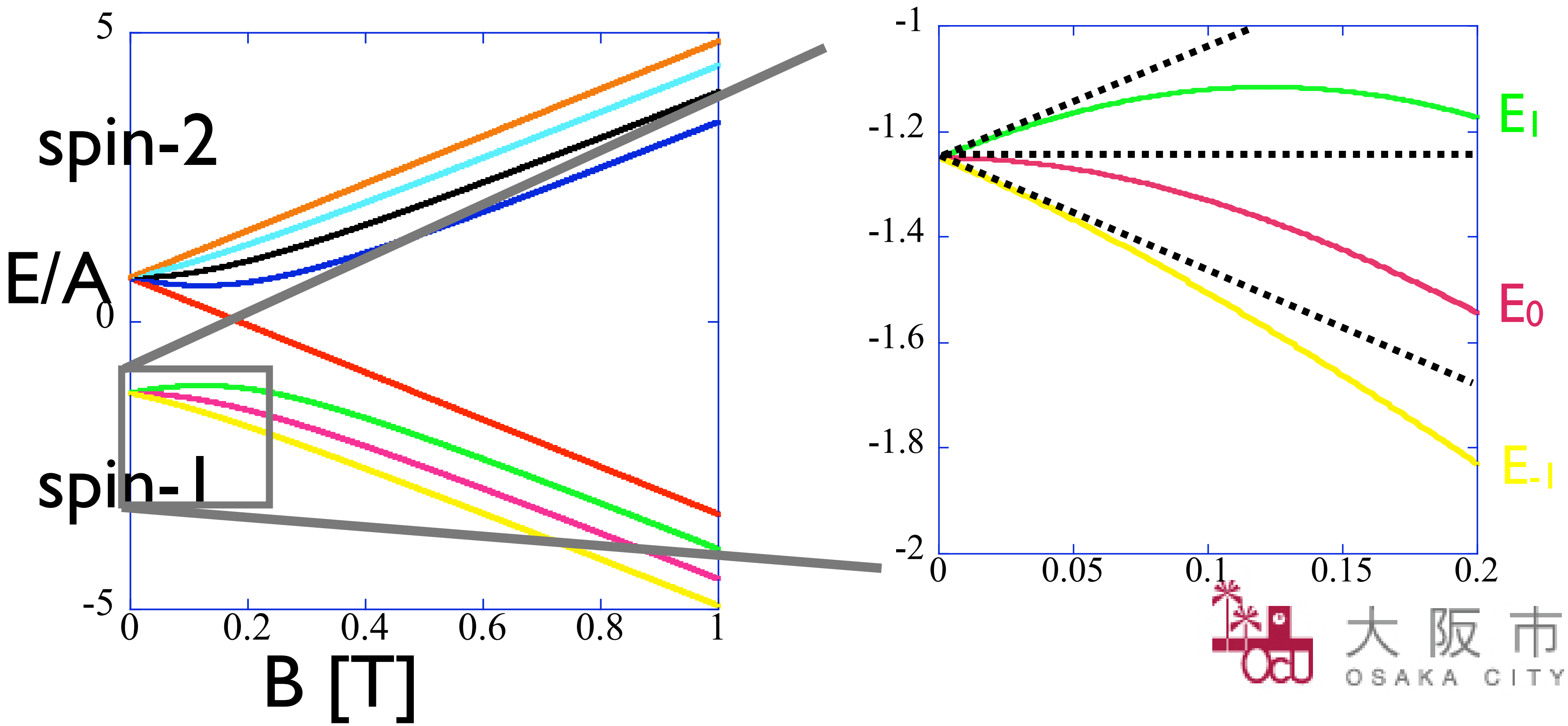
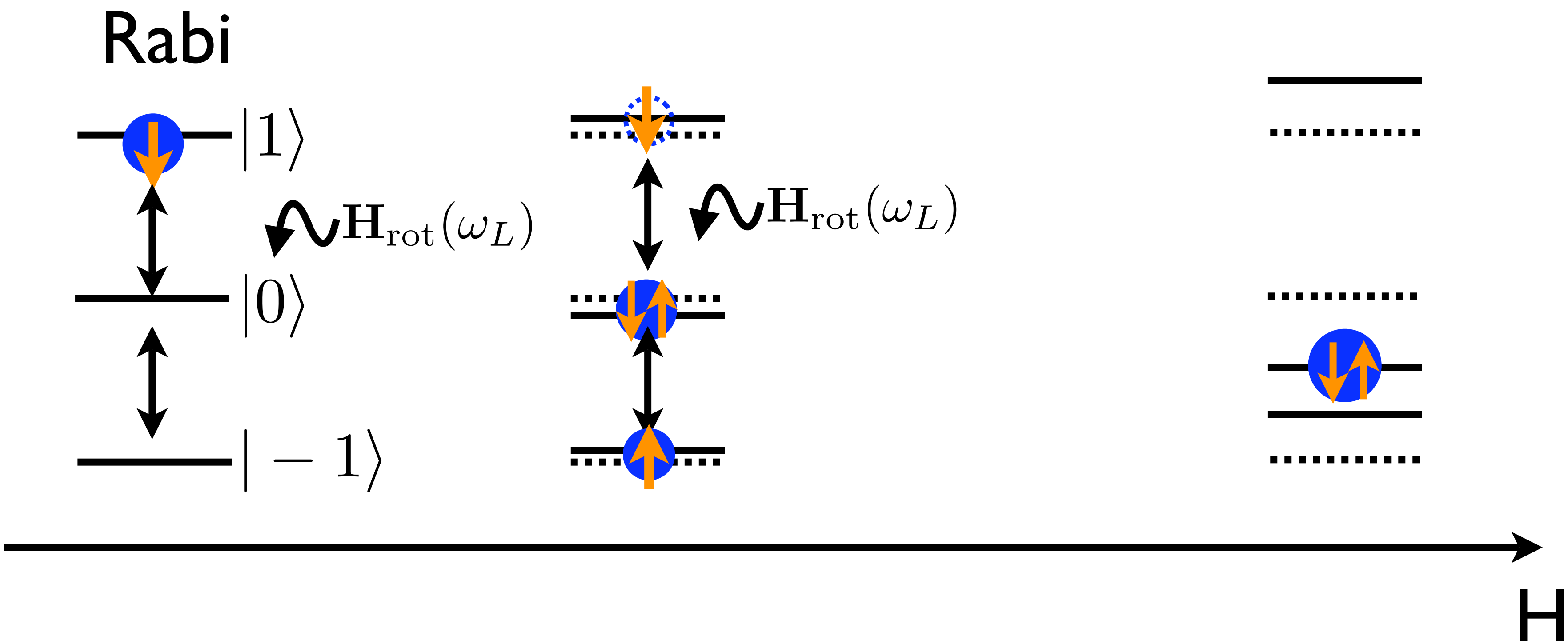
$$\delta = E_1 + E_{-1} - 2E_0 \propto B^2$$

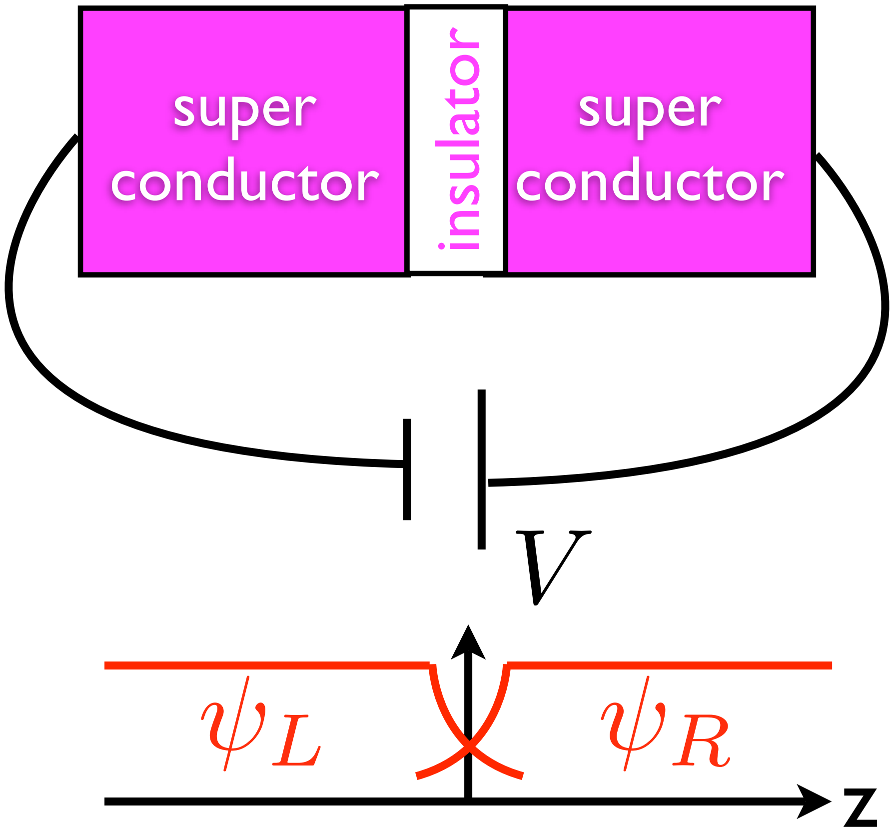


Rabi Transition



Josephson effects

JEs were predicted by Josephson in 1962[5], investigated by P.W. Anderson *et. al.* in 1963[6].



[5] B. D. Josephson Phys. Lett. **1,** 251-253 (1962)

[6] P.W. Anderson and J. M. Rowell, Phys. Rev. Lett. **10,** 230-232(1963)

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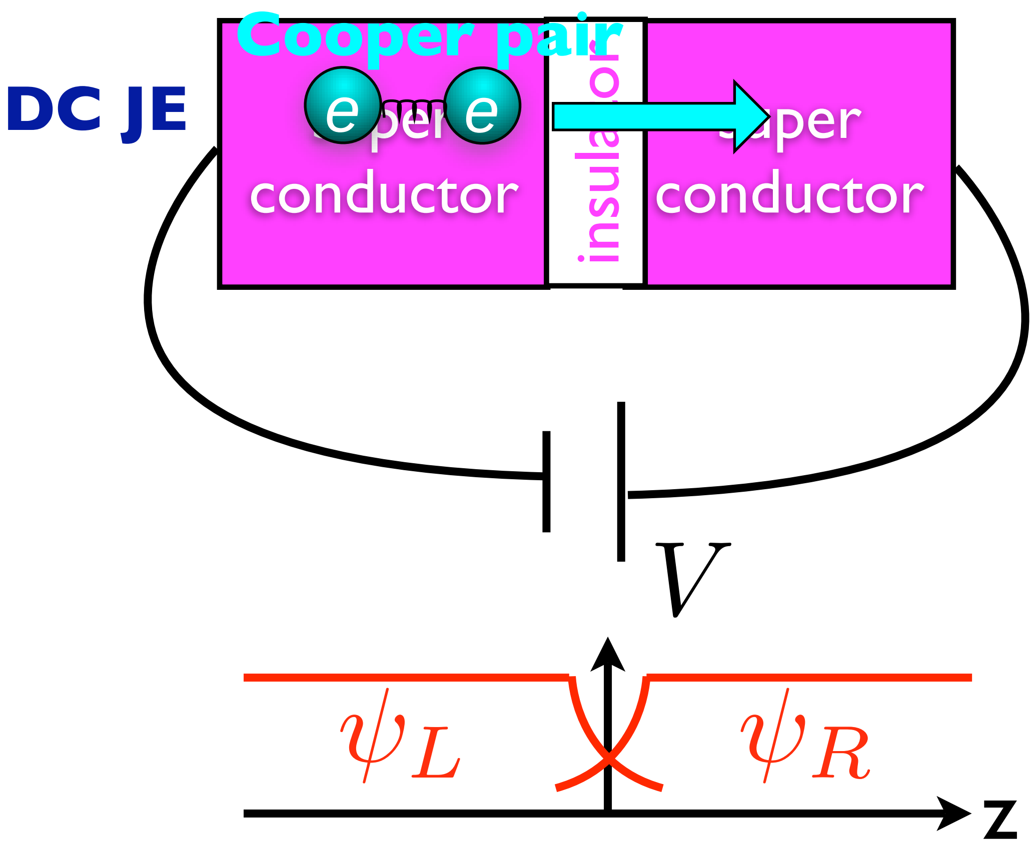

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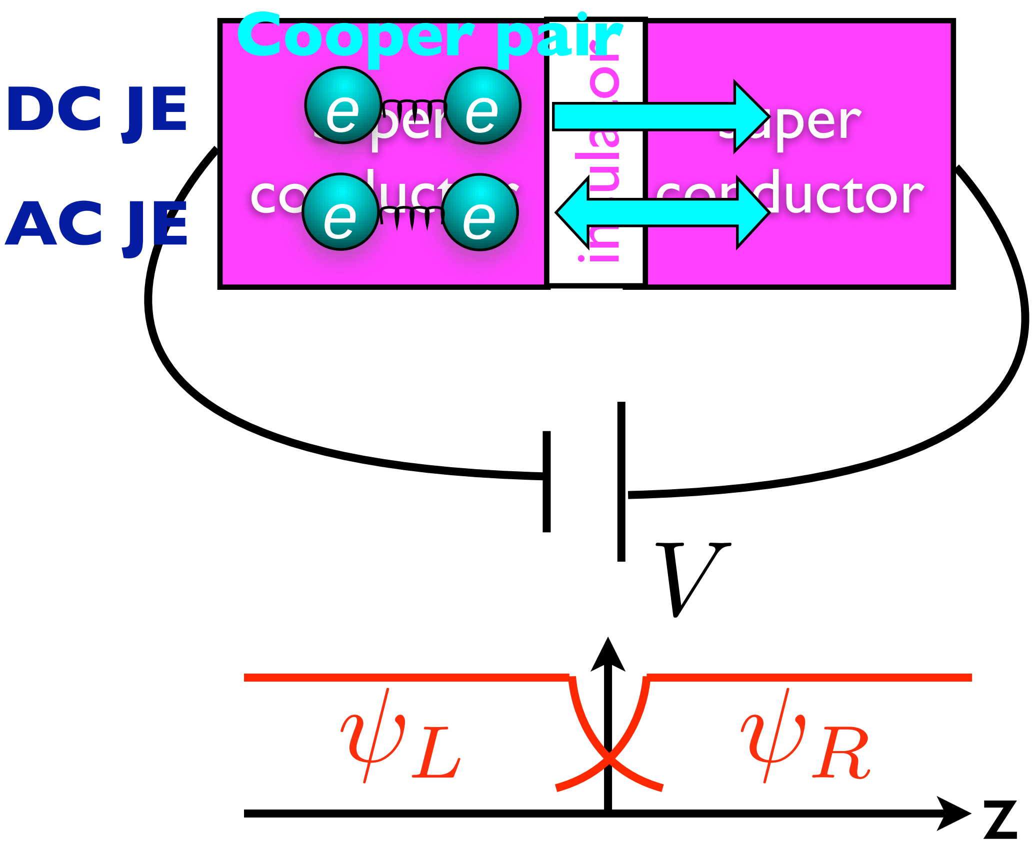

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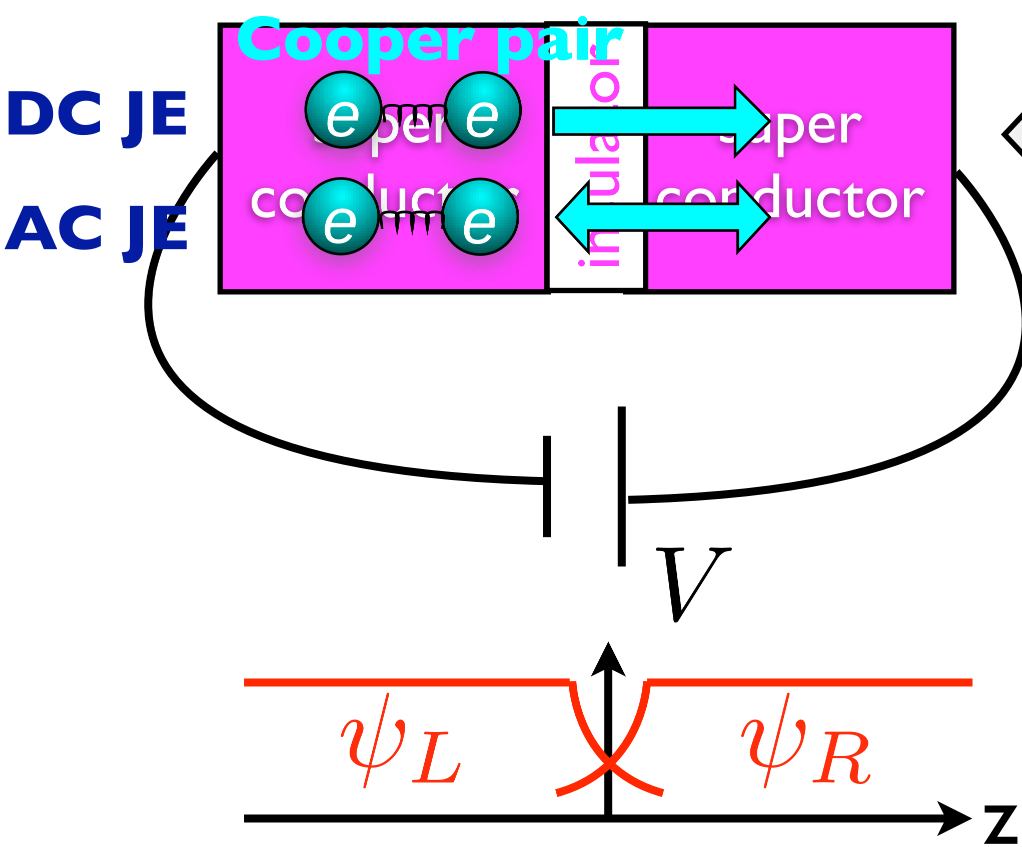
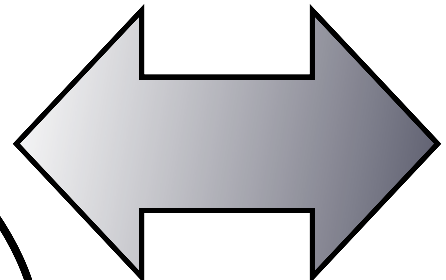
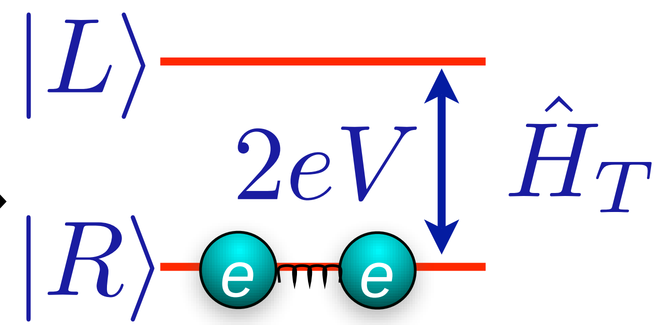




Feynman's discussion[7]



Josephson equations

$$J = J_1 \sin\phi$$

$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar}$$

$$\phi = \phi_R - \phi_L$$

[5] B. D. Josephson Phys. Lett. **1,** 251-253 (1962)
[6] P.W. Anderson and J. M. Rowell, Phys. Rev. Lett. **10,** 230-232(1963)
[7] R. P. Feynman, *et al., "The Feynman Lectures on Physics."* Vol. III (Addison-Welsey, 1965) chap. 21

Josephson type equation[8]

$$\mathbf{H} = H\hat{z}$$

$$i\hbar\frac{d}{dt}|\xi\rangle = (\hat{H}_{lZ} + \hat{H}_{qZ} + \hat{H}_{int})|\xi\rangle$$

$$\dot{\rho}_0 = \frac{2c}{\hbar}\rho_0\sqrt{(1-\rho_0)^2 - m^2}\sin\theta$$

$$\dot{\theta} = -\frac{2\delta}{\hbar} + \frac{2c}{\hbar}(1-2\rho_0) + \frac{2c}{\hbar}\frac{(1-\rho_0)(1-2\rho_0)-m^2}{\sqrt{(1-\rho_0)^2-m^2}}\cos\theta$$

magnetization $m = (N_1 - N_{-1})/N$

relative phase $\theta = \theta_1 + \theta_{-1} - 2\theta_0$

relative Zeeman ene. $\underline{\delta = E_1 + E_{-1} - 2E_0 \propto H^2}$

interaction coefficient $c = c_2 N \int |\phi| d\mathbf{r}$

[8] W. Zhang *et al.*, Phys. Rev. A **72**, 013602 (2005).

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AC JEs in a spinor BEC

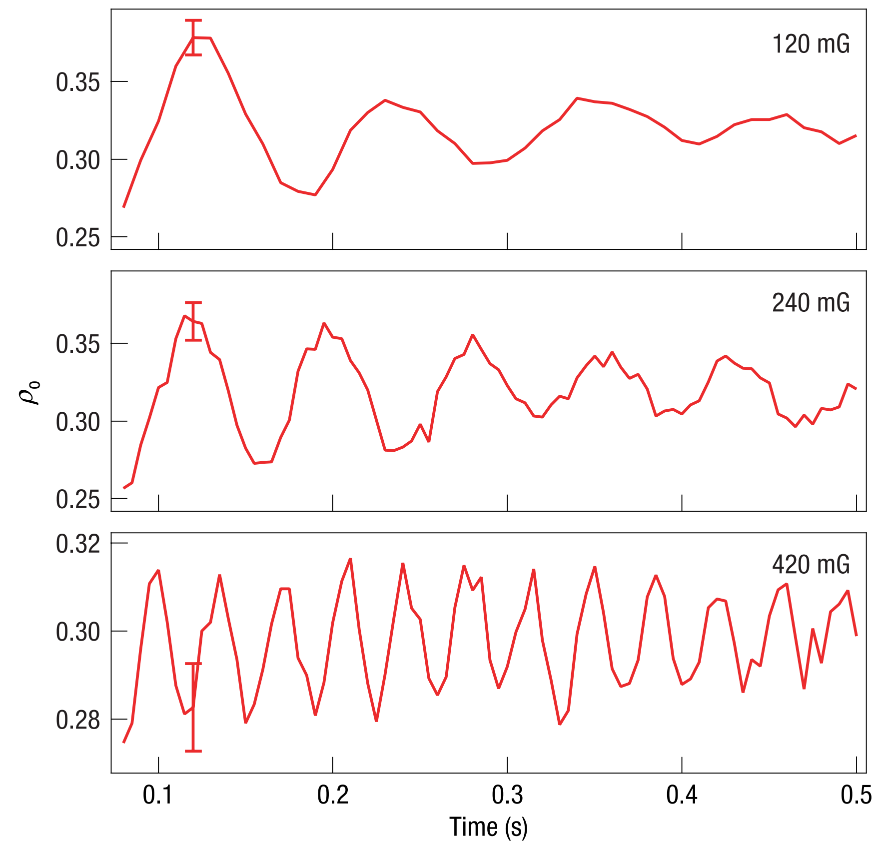
In the limit $\delta \gg c$

AC Josephson effects

$$\rho_0(t) \propto \frac{1}{\delta} \sin\left(\frac{2\delta t}{\hbar}\right)$$

$$\dot{\theta} \sim -\frac{2\delta}{\hbar}$$

the experimental results[9]



[9] M. -S. Chang, et. al, Nature, Phys. 1, 111(2005).

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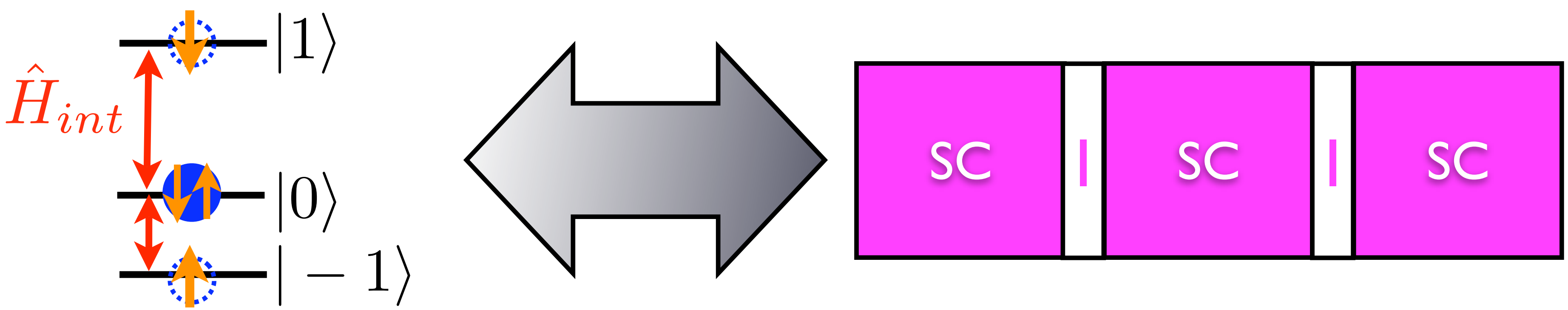
Internal JEs

As the Feynman's discussion, we can consider the three level system to be junctions of three super conductors.



Maki-Tsuneto discussed internal JEs in ^{3}He-A[10], and Whetly obtained the effects experimentally[11].

$$|\uparrow\uparrow\rangle \overset{\hat{H}_{dd}}{\longleftrightarrow} |\downarrow\downarrow\rangle$$

[10] K. Maki and T. Tsuneto, Prog. Theor. Phys. **52,** 774 (1974)
[11] R. A. Webb, *et al.*, Phys. Lett. **48A,** 421(1974)

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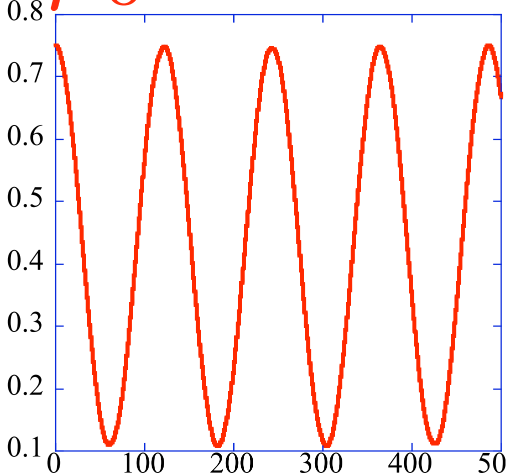
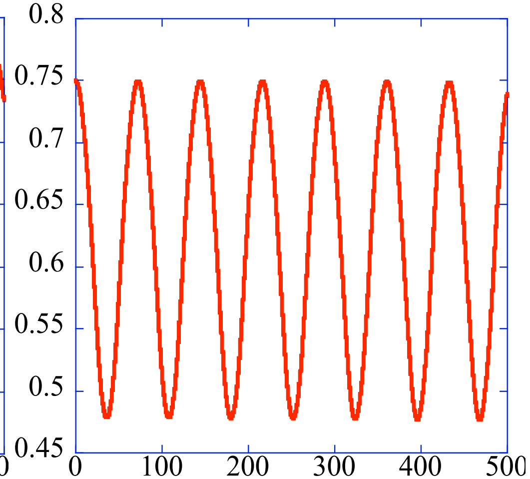
Numerical GP solutions of JEs

ρ_0 H = 0

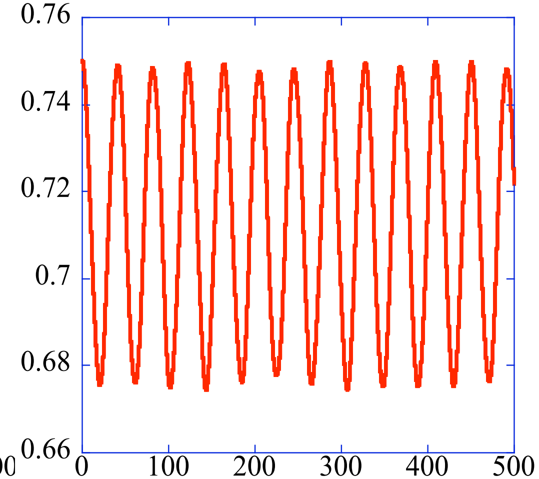


time

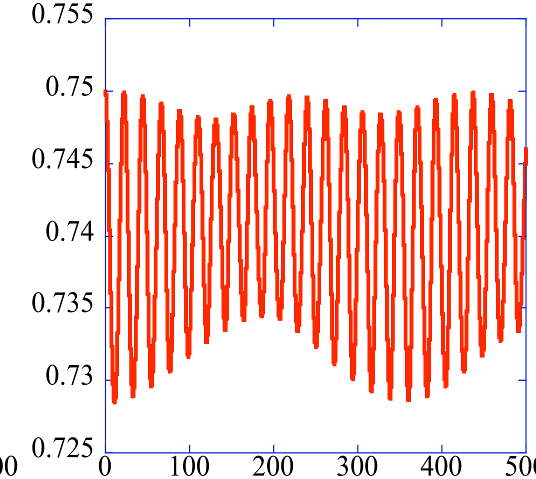
H=H0



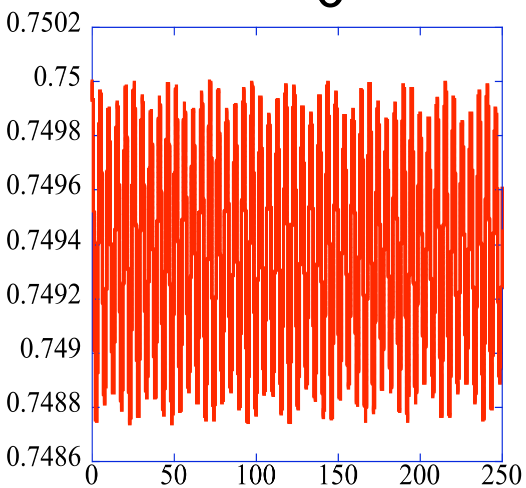
1.5H0



2H0



4H0



initial population

$$(\rho_1(0), \rho_0(0), \rho_{-1}(0)) = (0, 0.75, 0.25)$$

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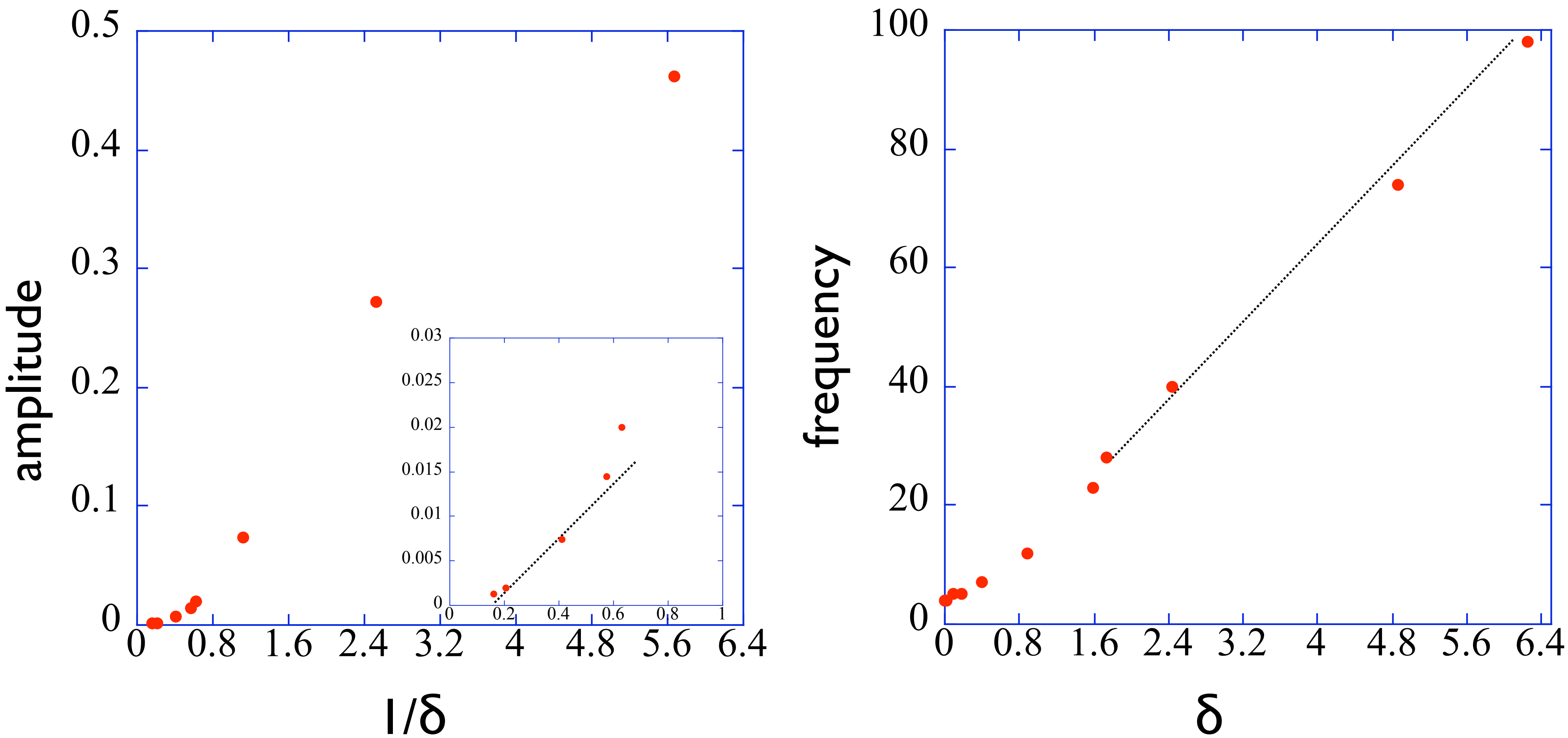
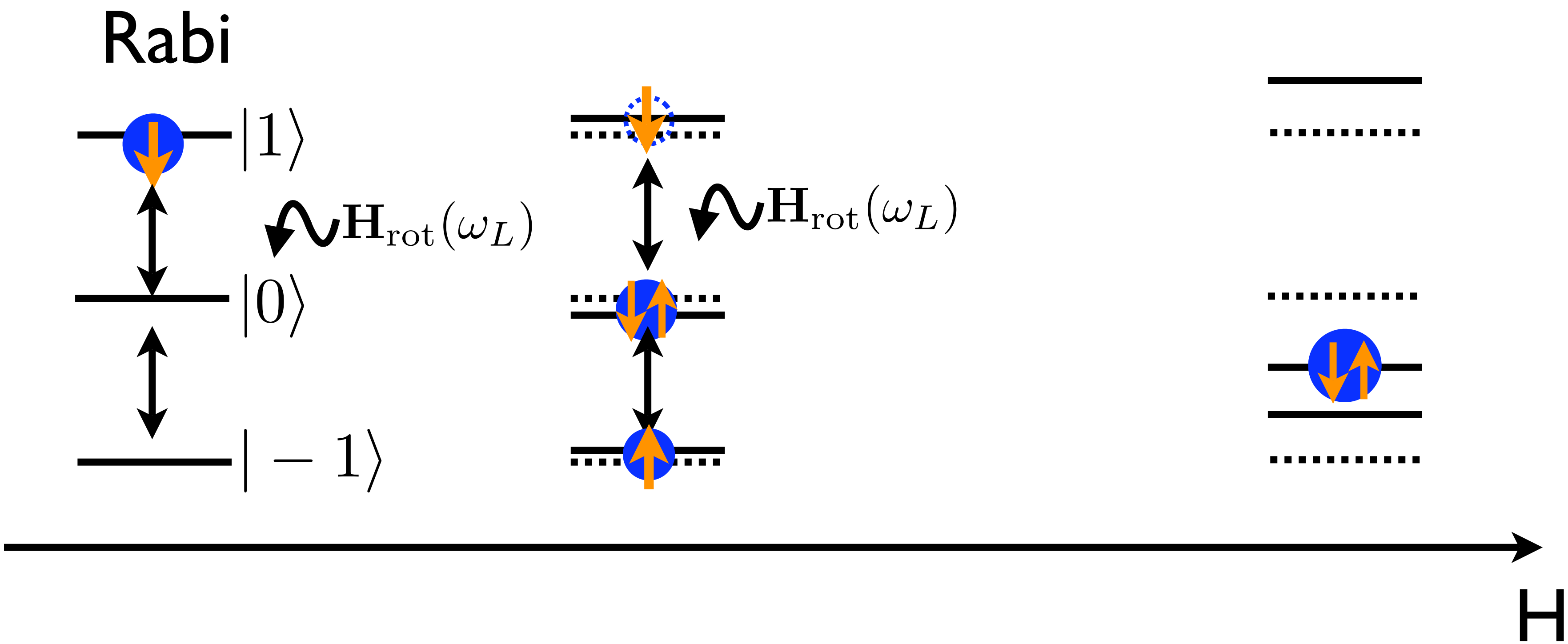
Numerical GP solutions of JEs

In SMA and the limit $\delta \gg c$

$$\rho_0(t) \propto \frac{1}{\delta} \sin\left(\frac{2\delta t}{\hbar}\right) \qquad \delta \propto H^2$$



Rabi oscillation
+ quadratic Zeeman effects

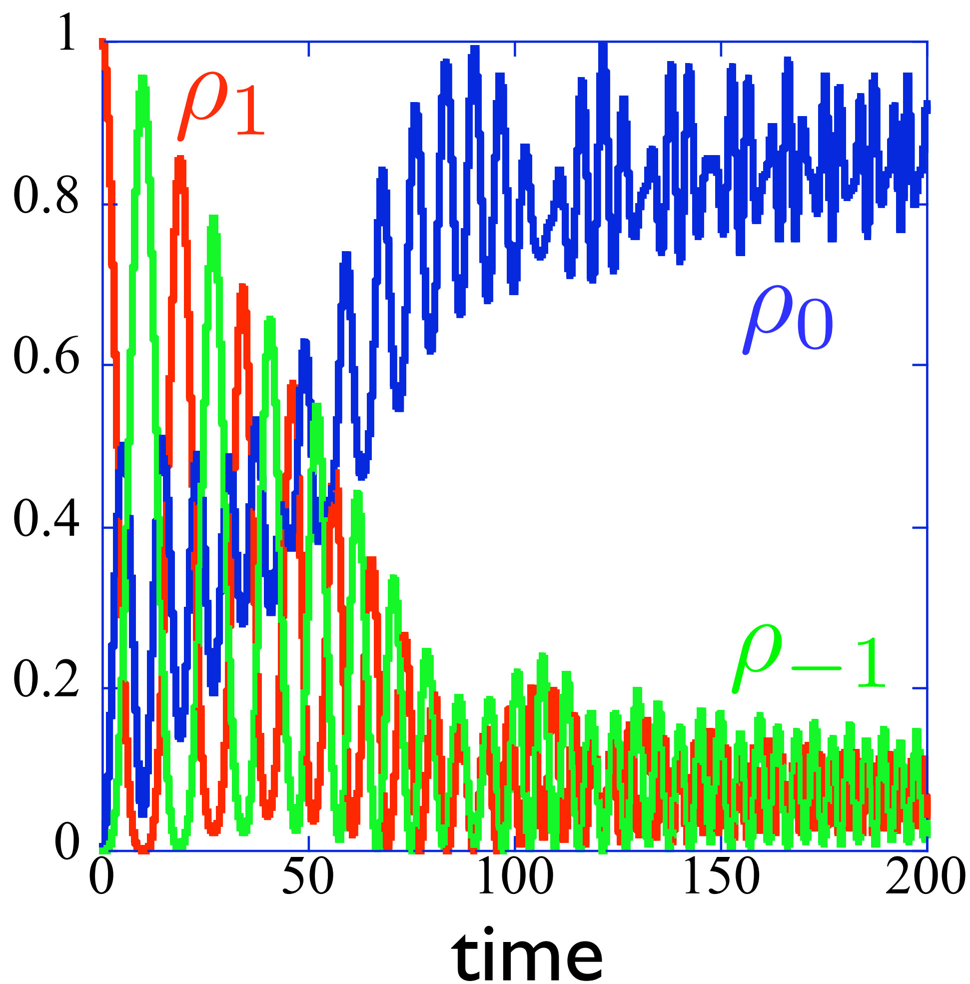


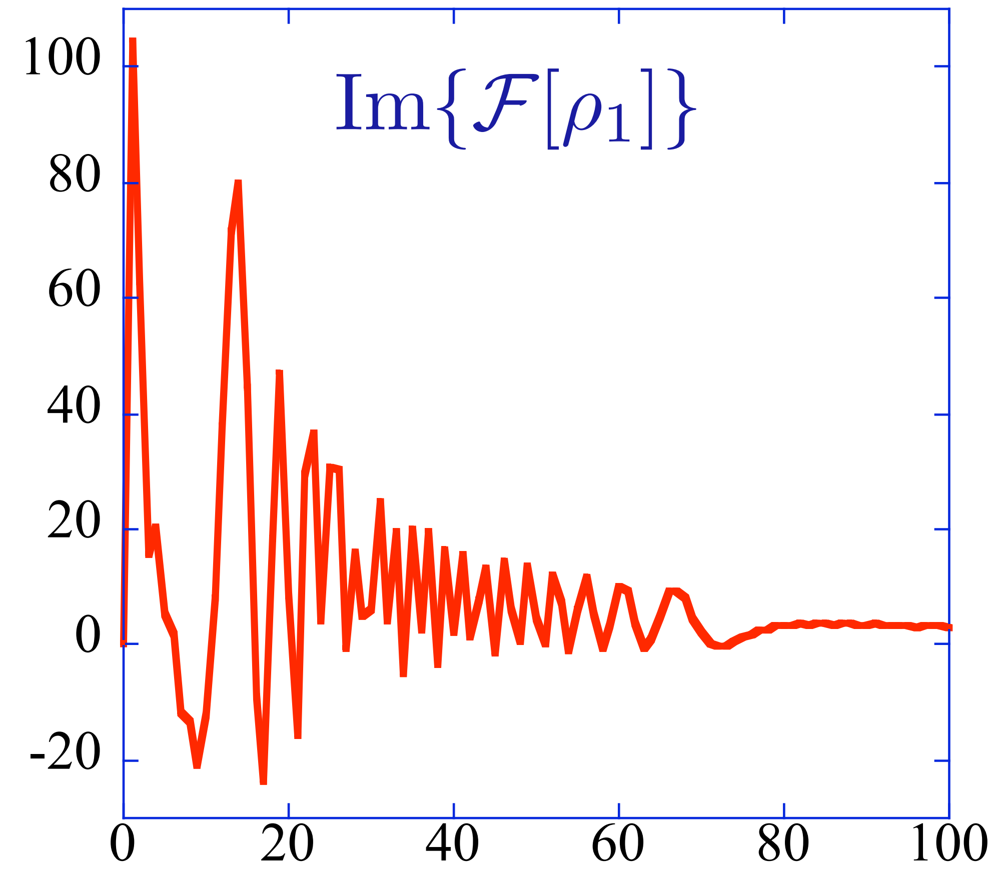
Transition from Rabi to Josephson

$$\mathbf{H} = H_1 \cos\omega(t)t\hat{\mathbf{x}} - H_1 \sin\omega(t)t\hat{\mathbf{y}} + H(t)\hat{\mathbf{z}}$$

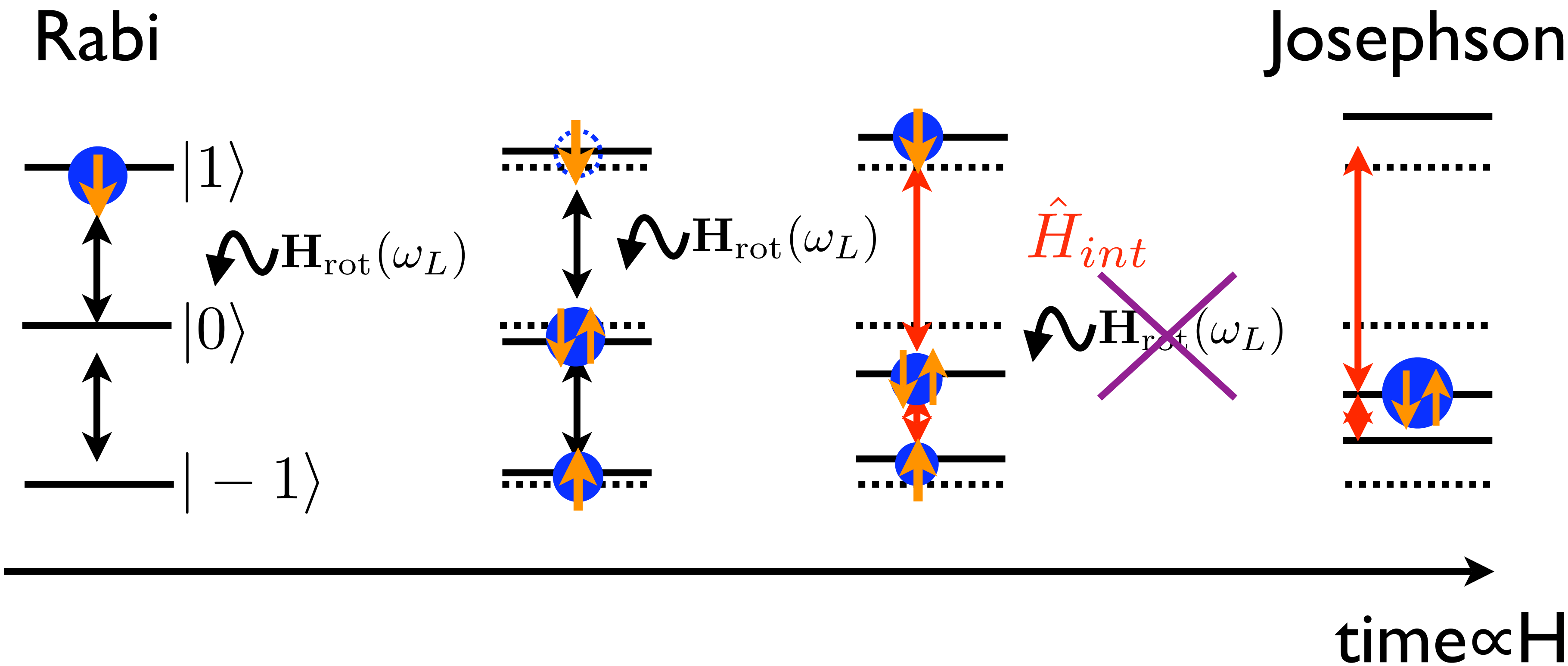
$$H(t) = at + h_0$$
$$\omega(t) = \gamma H(t)$$

initial population $(1, 0, 0)$





Namely..


Rabi
Josephson
$|1\rangle$
$|0\rangle$
$|-1\rangle$
$\mathbf{H}_{\mathrm{rot}}(\omega_L)$
$\mathbf{H}_{\mathrm{rot}}(\omega_L)$
$\hat{H}_{int}$
$\mathbf{H}_{\mathrm{rot}}(\omega_L)$
time∝H

Summary and Future

- We obtain Internal Josephson effects by calculating GP equation directly.
- We obtained the transition from Rabi to Josephson.
- We will discuss effects of MDDI in ferromagnetic resonance

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