Supercomputing tool for superfluid systems

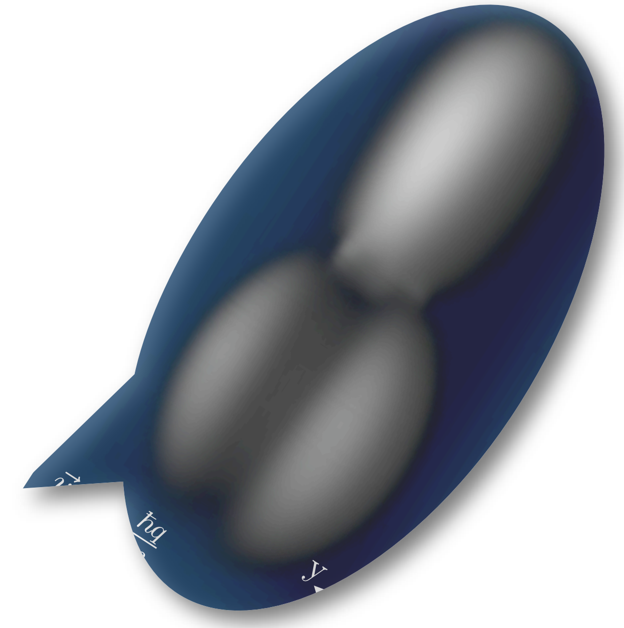
Tapio Simula and Kazushige Machida

岡山大学

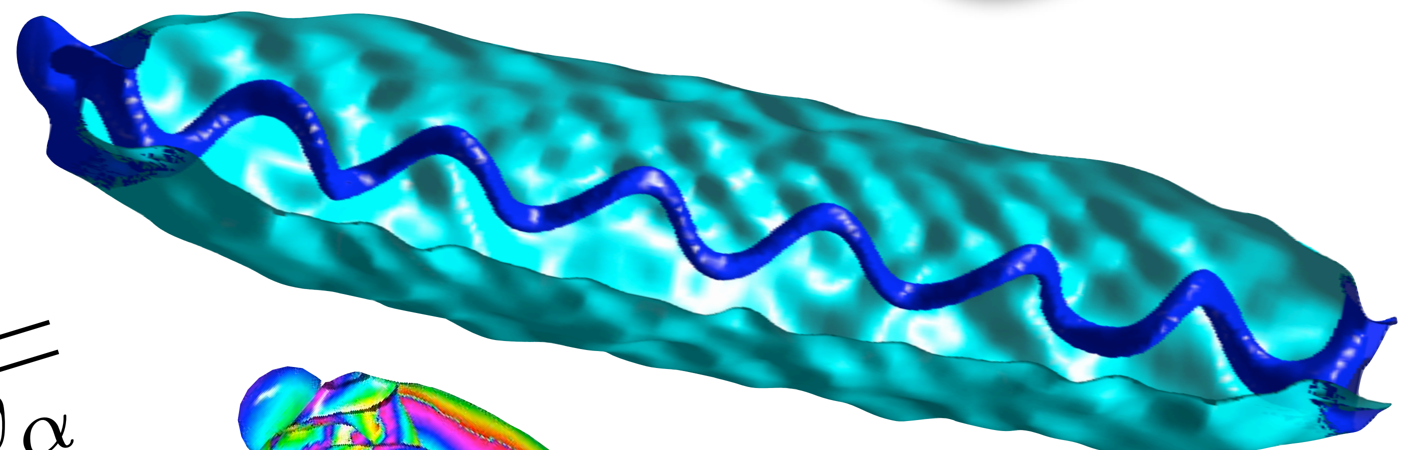
Outline

✓ Solved problems

- Bragg scattering a vortex state



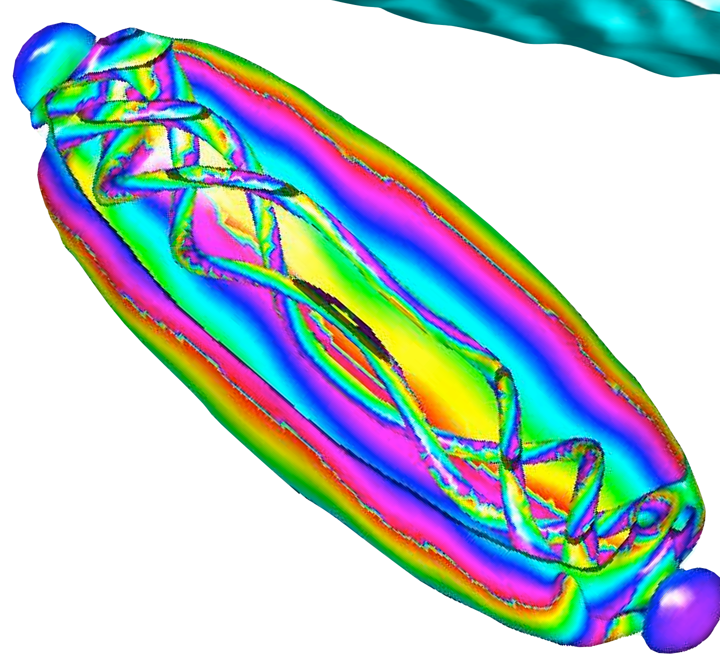
- Vortex waves



✓ Methodology

$$u_\alpha(x_\beta) = \frac{\delta_{\alpha\beta}}{\sqrt{w_\alpha}}$$

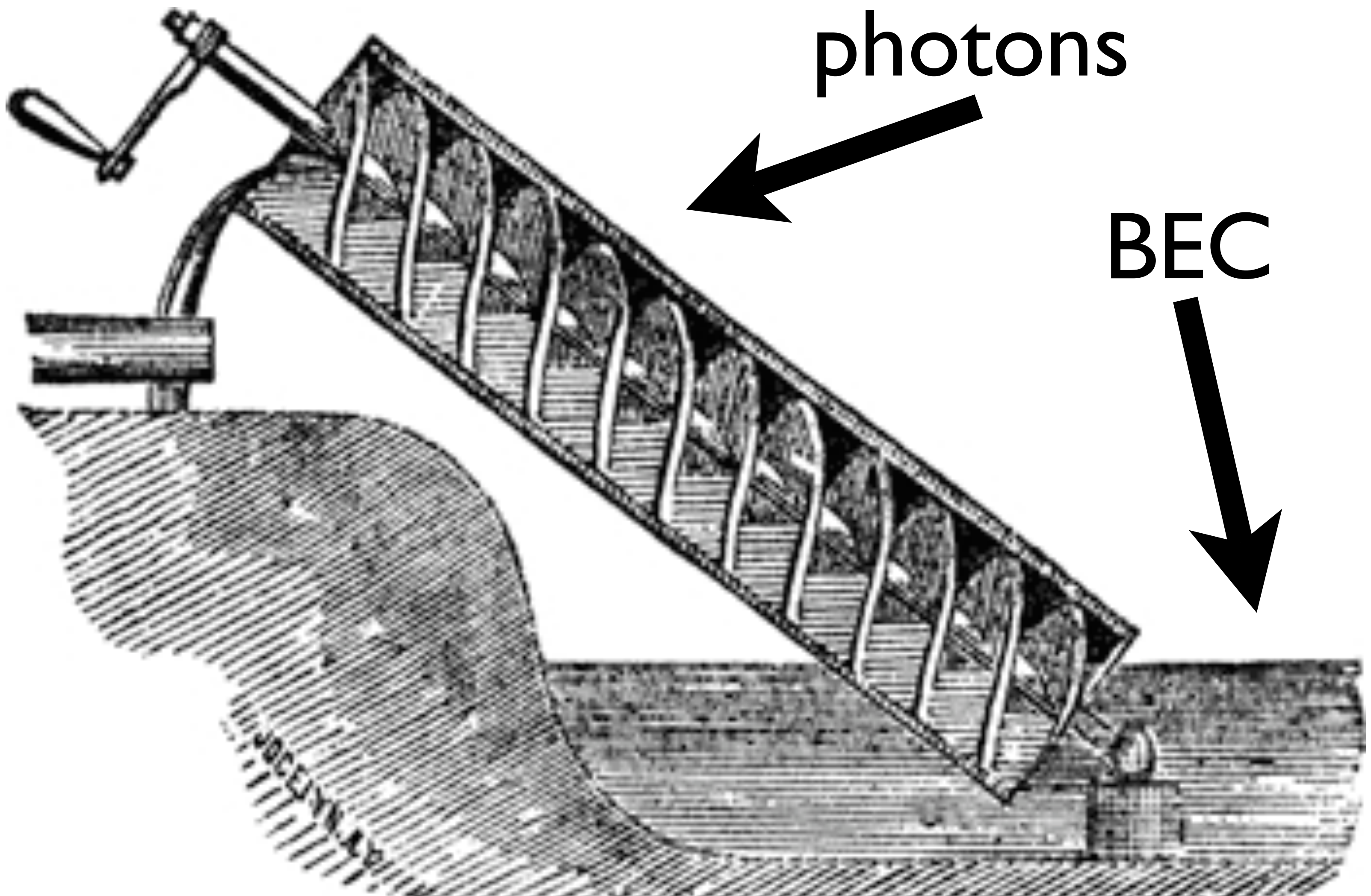
✓ Problems to be solved

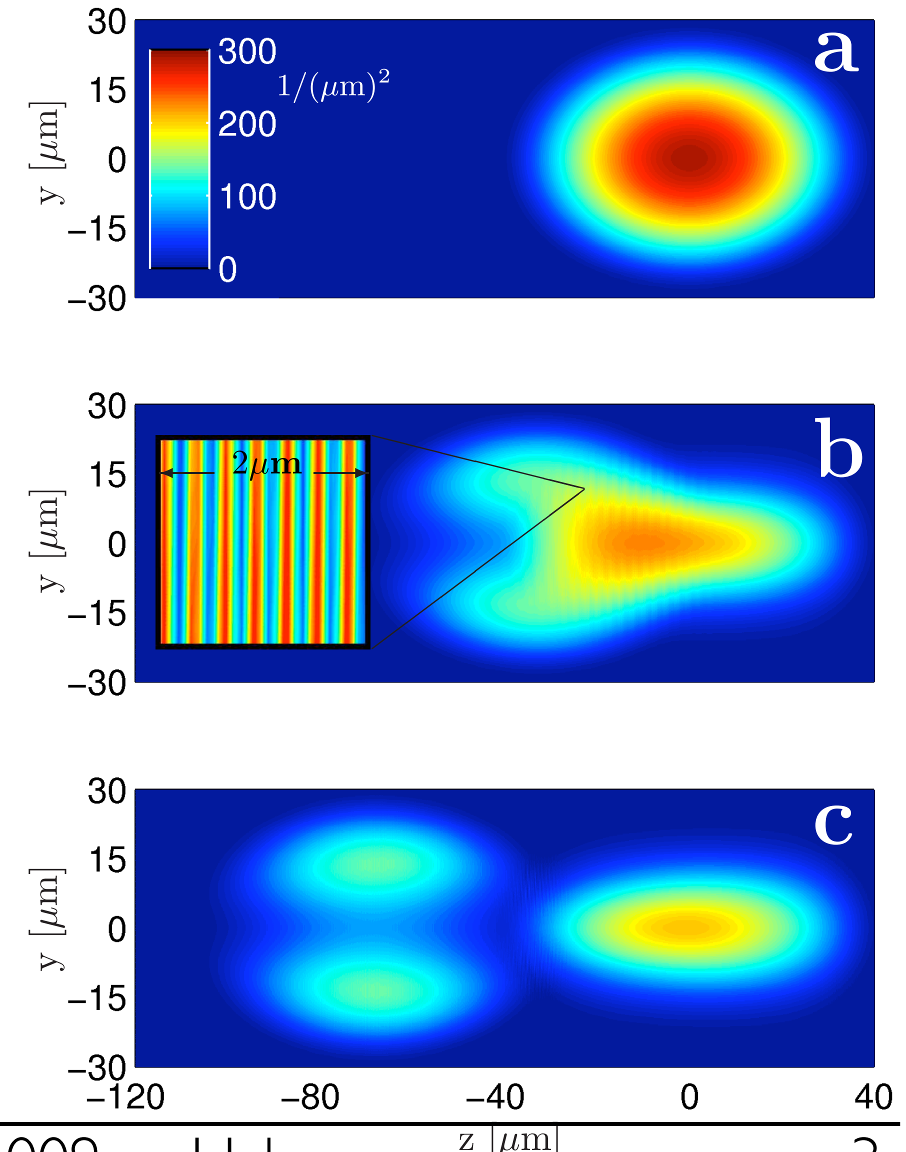


Archimede's quantum screw

light-shift potential from Laguerre+Gaussian laser fields

$$V_{\mathrm{ext}}(\mathbf{r},t) = |A_{\mathrm{G}}|^2 + |A_{\mathrm{LG}}|^2 + 2A_{\mathrm{G}}^{*}A_{\mathrm{LG}}\cos(2kz + \Delta\omega t + \phi)$$



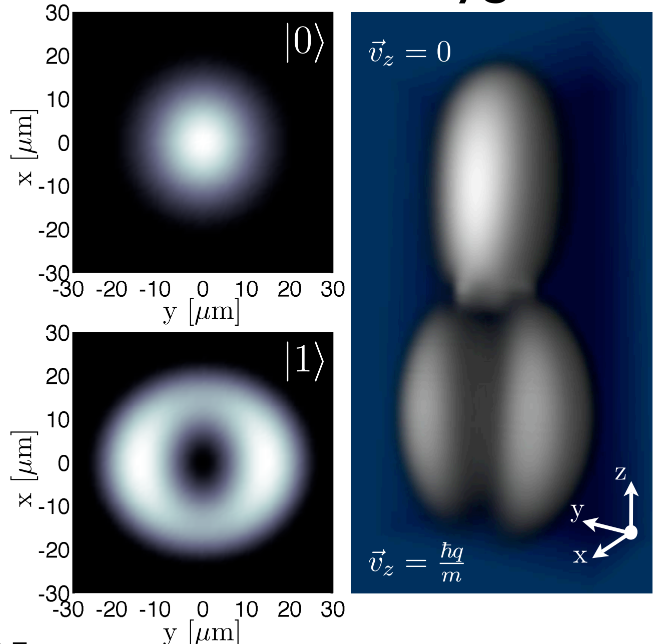


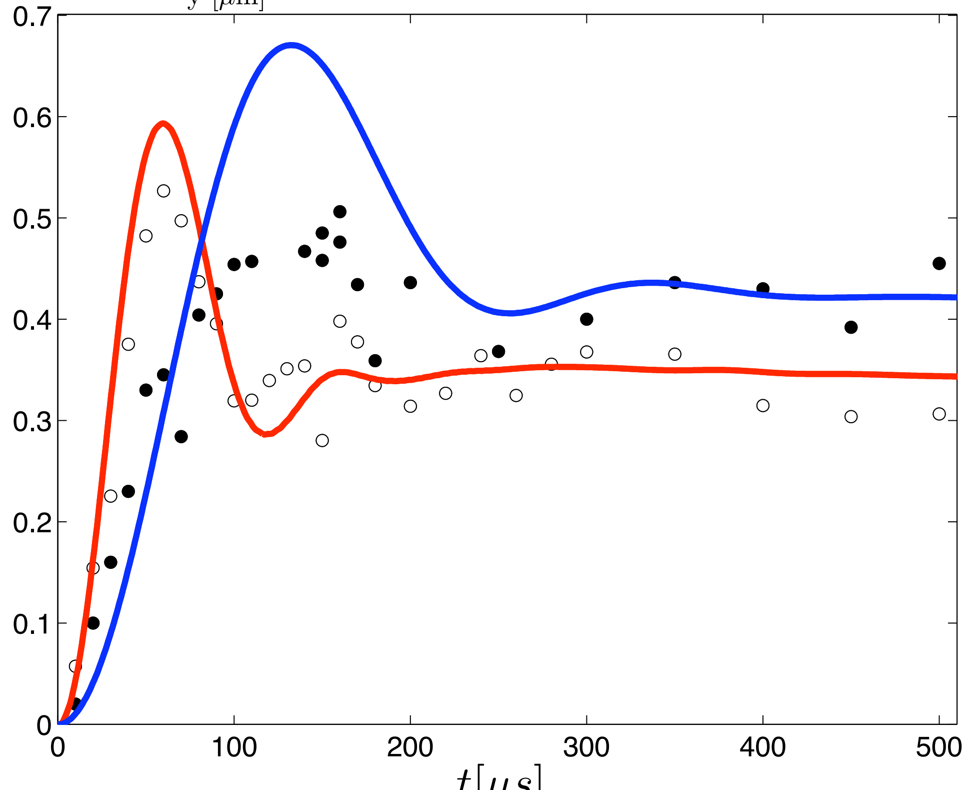
PRL 97, 170406 (2006) PHYSICAL REVIEW LETTERS week ending 27 OCTOBER 2006

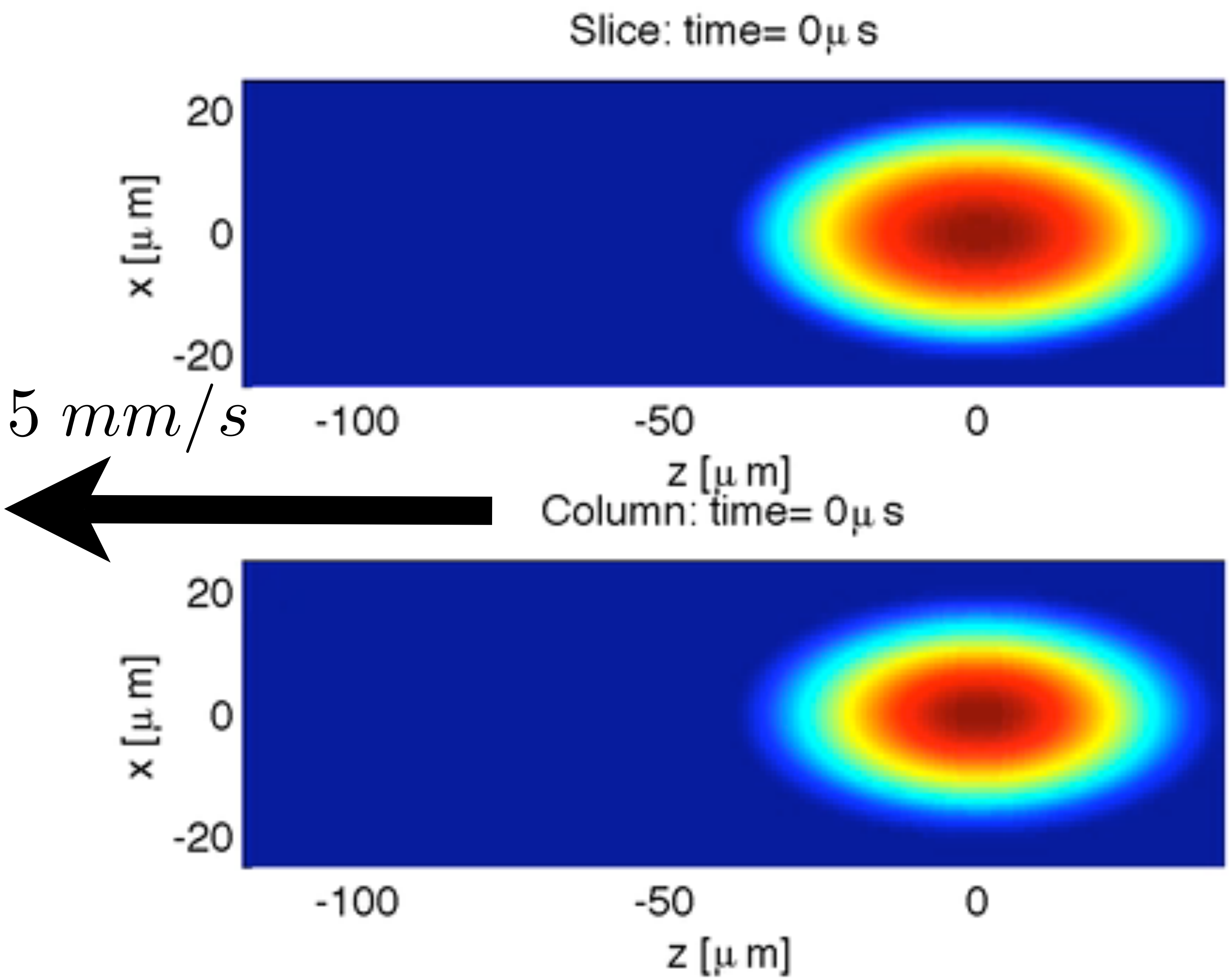
Quantized Rotation of Atoms from Photons with Orbital Angular Momentum

M. F. Andersen, C. Ryu, Pierre Cladé, Vasant Natarajan,* A. Vaziri,† K. Helmerson, and W. D. Phillips
Atomic Physics Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8424, USA
(Received 26 June 2006; published 26 October 2006)

T. P. Simula, N. Nygaard, S. X. Hu, L. A. Collins, B. I. Schneider, and K. Mølmer, Phys. Rev. A 77, 015401 (2008)

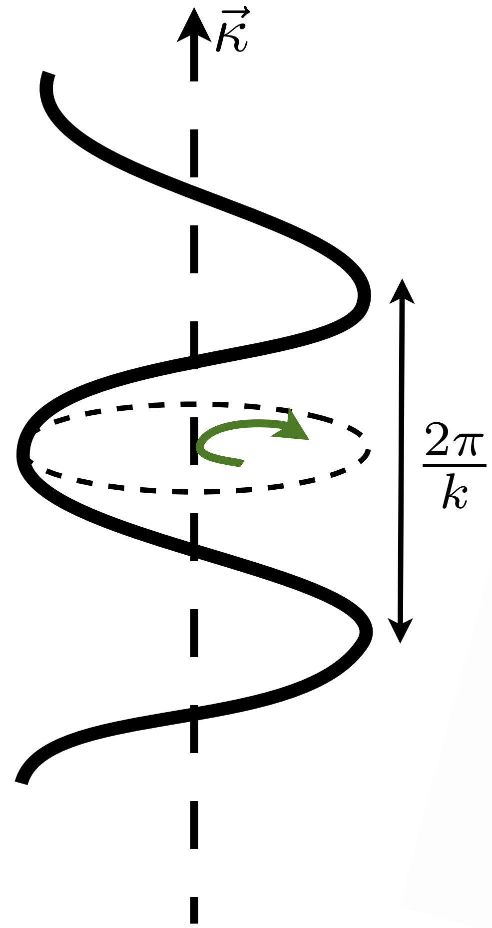


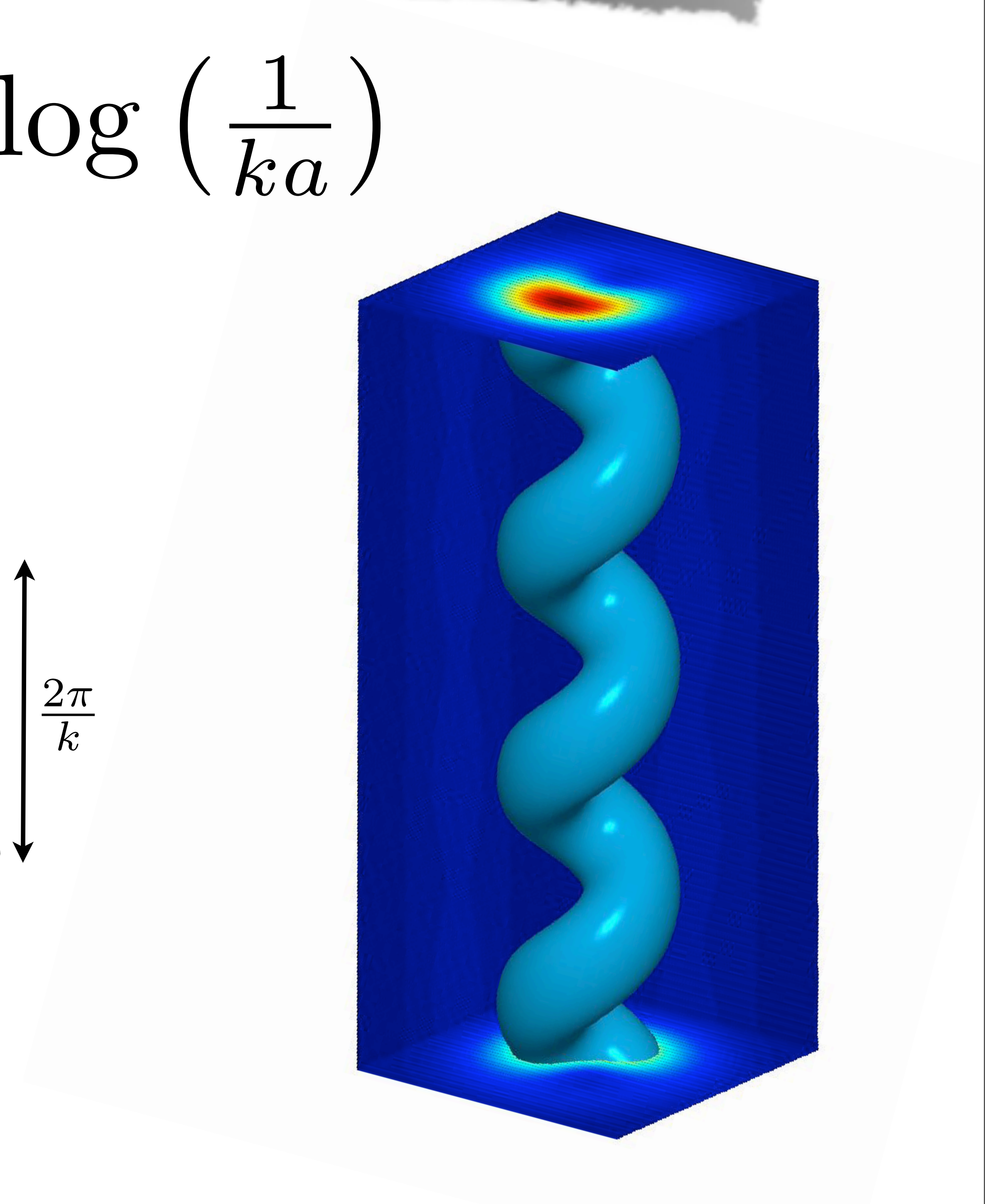




Kelvin waves

$$\omega(k) \approx \frac{\Gamma k^2}{4\pi} \log\left(\frac{1}{ka}\right)$$





VOLUME 90, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 14 MARCH 2003

Quadrupole Oscillation of a Single-Vortex Bose-Einstein Condensate: Evidence for Kelvin Modes

V. Bretin, P. Rosenbusch, F. Chevy, G.V. Shlyapnikov,* and J. Dalibard

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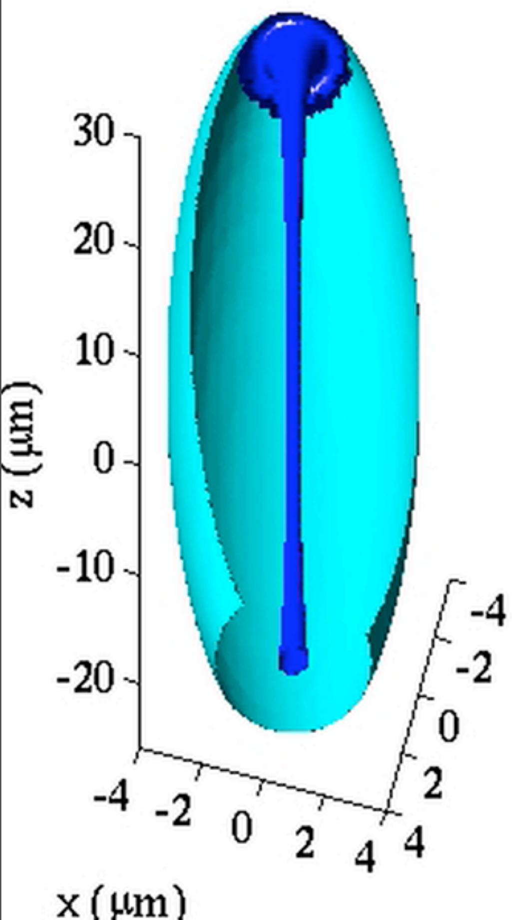
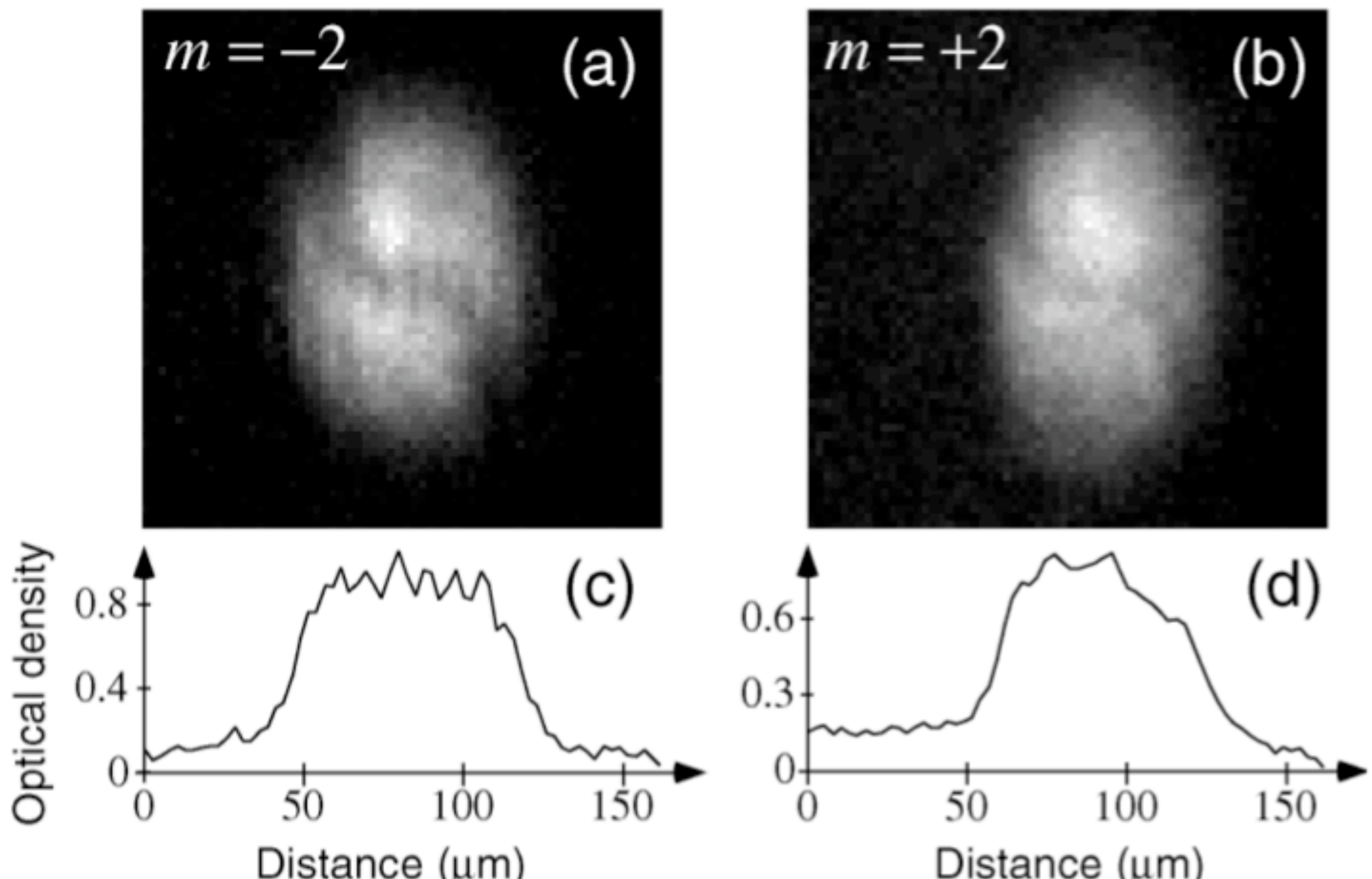
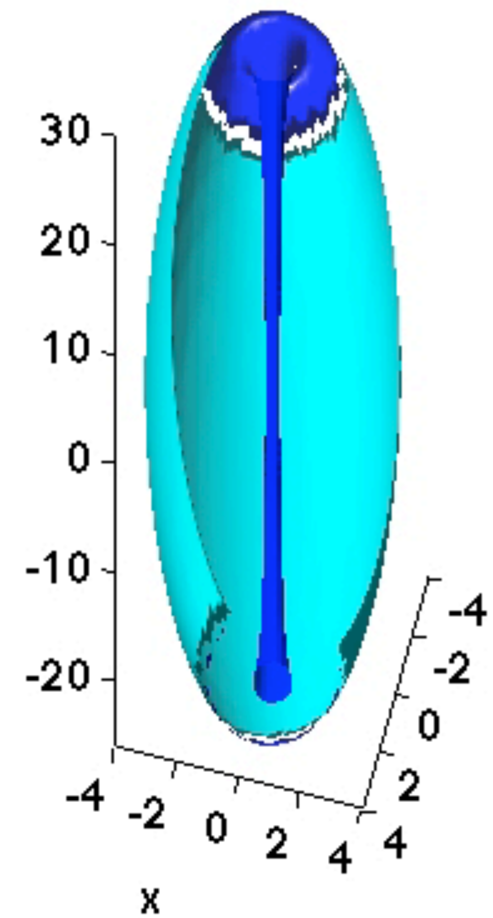
(Received 5 November 2002; published 12 March 2003)

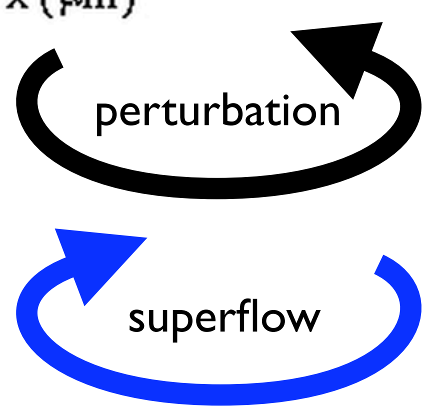
$m = -2$

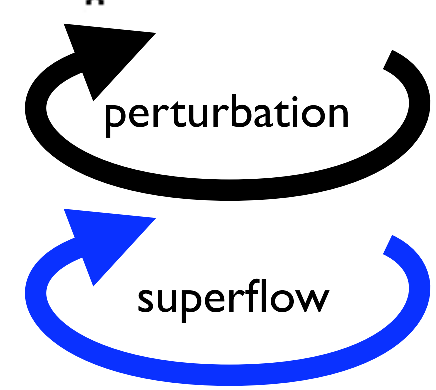
$m = +2$





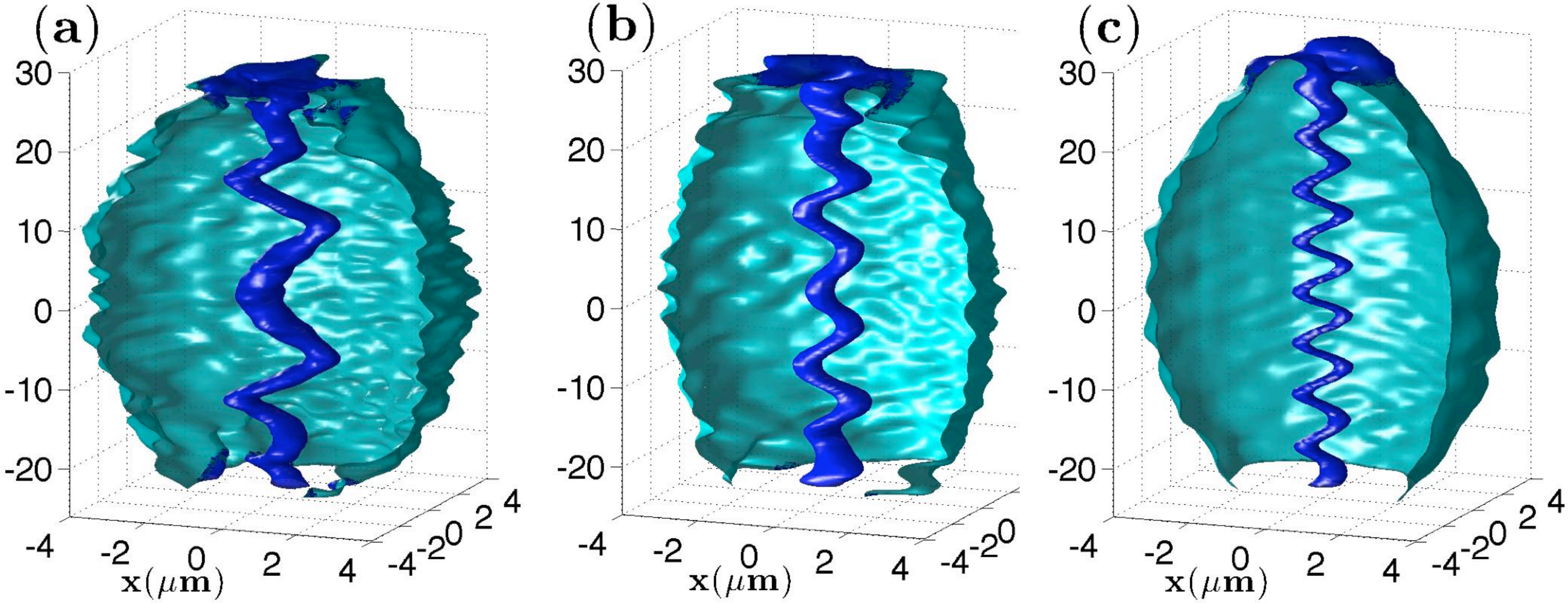


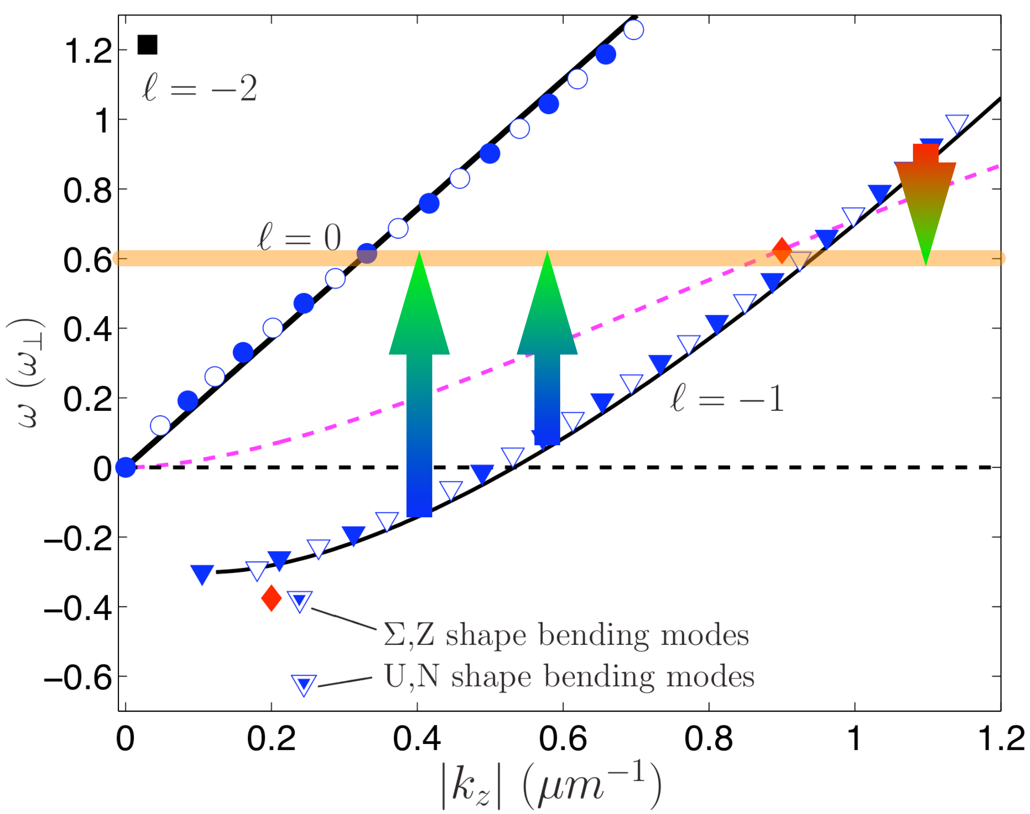




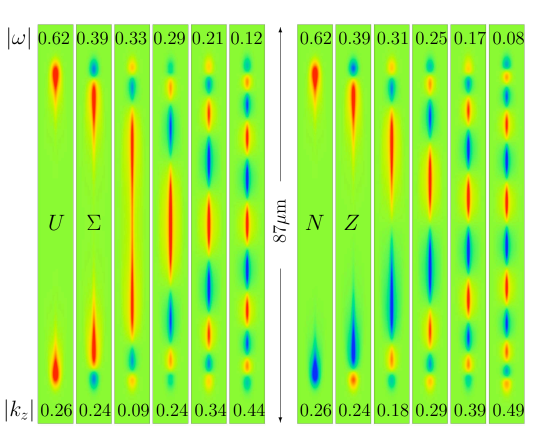
$$V(t) = \frac{1}{2} m\omega^2 \epsilon (X^2 - Y^2)$$

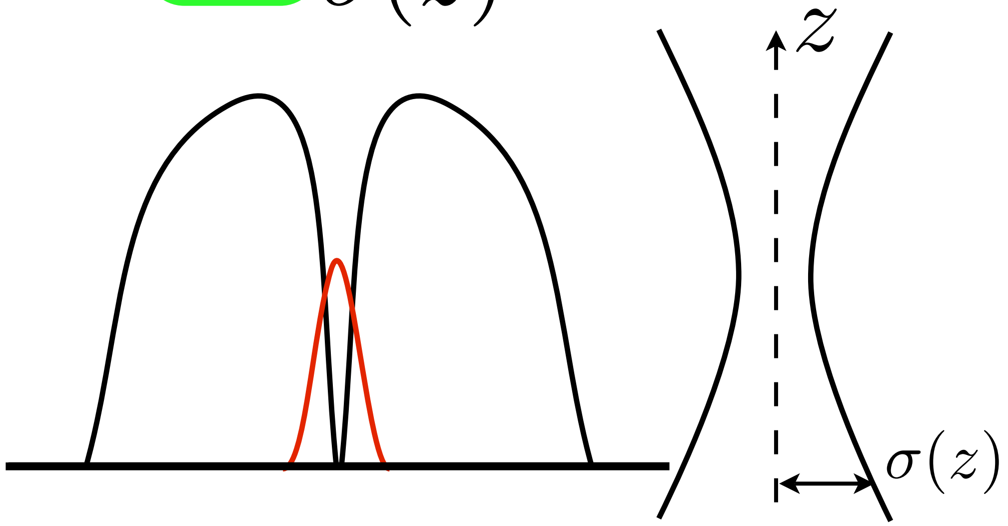
T. P. Simula, T. Mizushima, and K. Machida, PRL (2008)



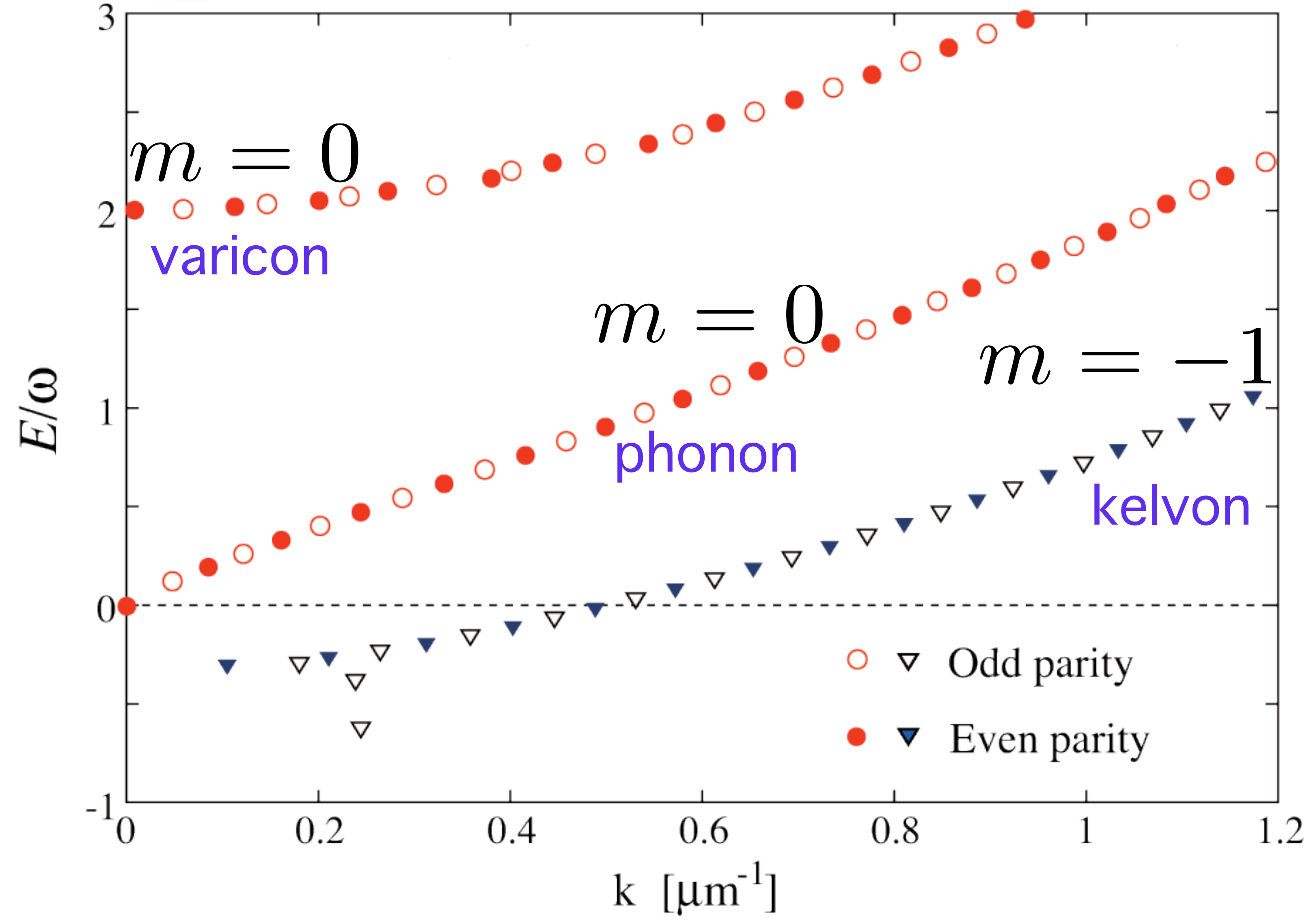


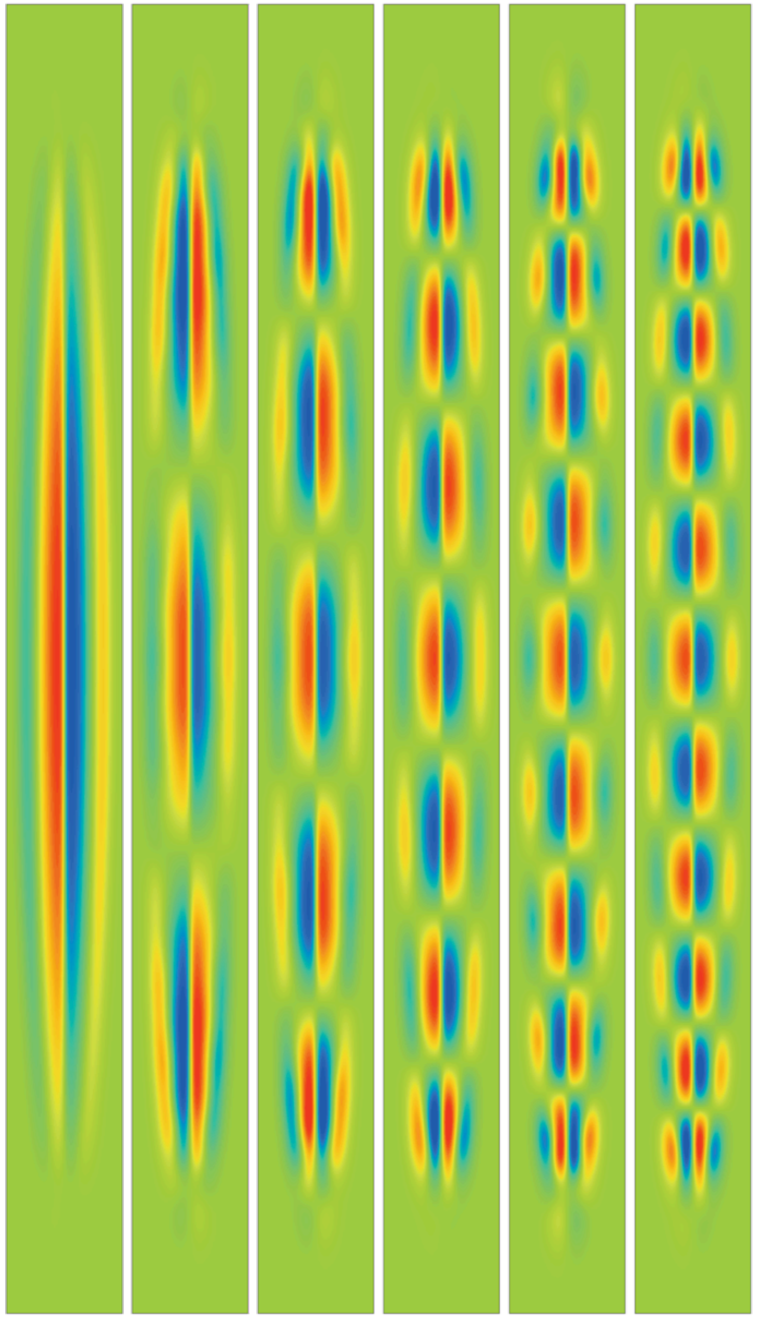
$$V_{\rm pin} = V_0 \frac{\sigma_0^2}{\sigma(z)^2} e^{-\frac{2r^2}{\sigma(z)^2}}$$





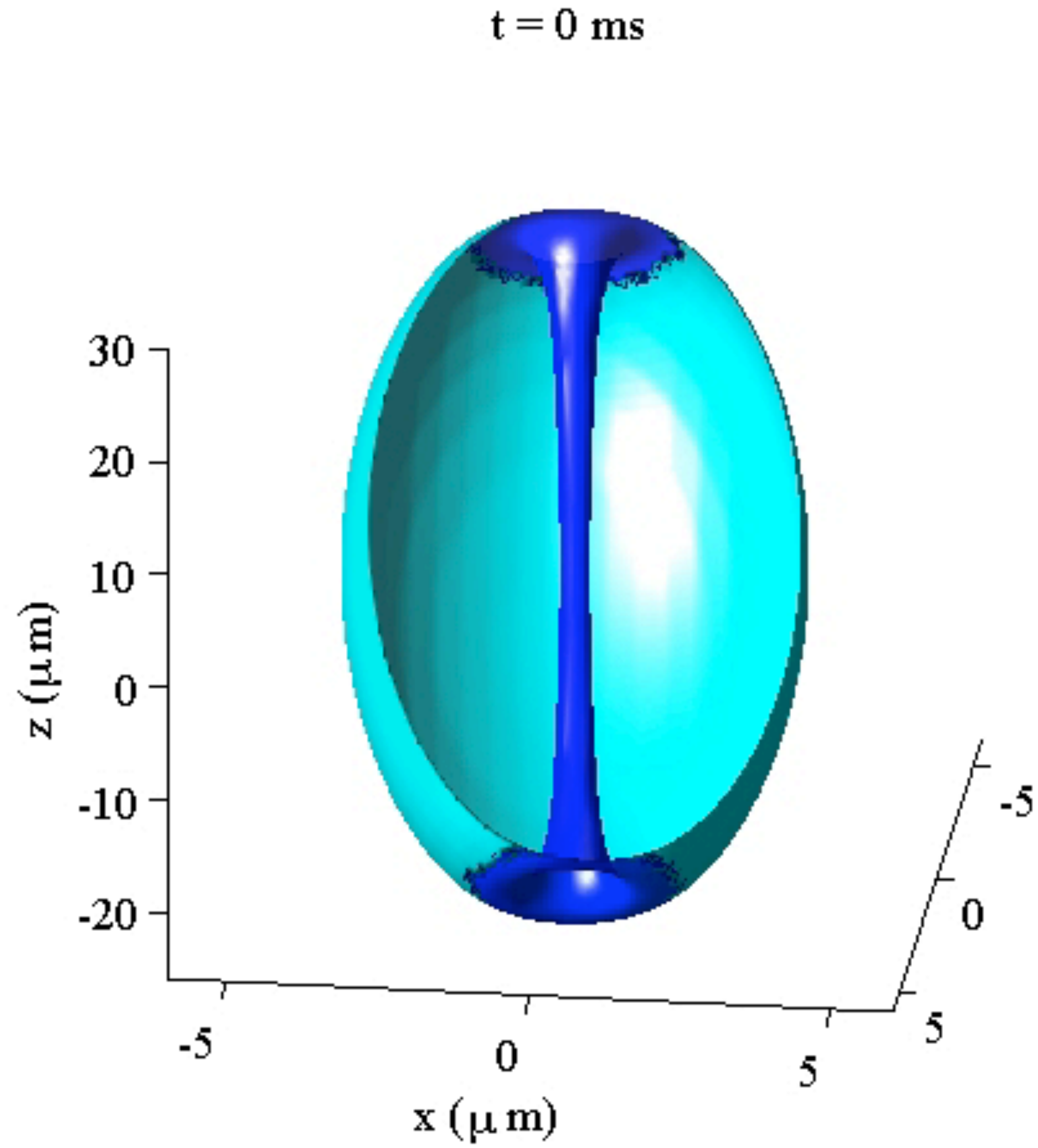
Varicose waves

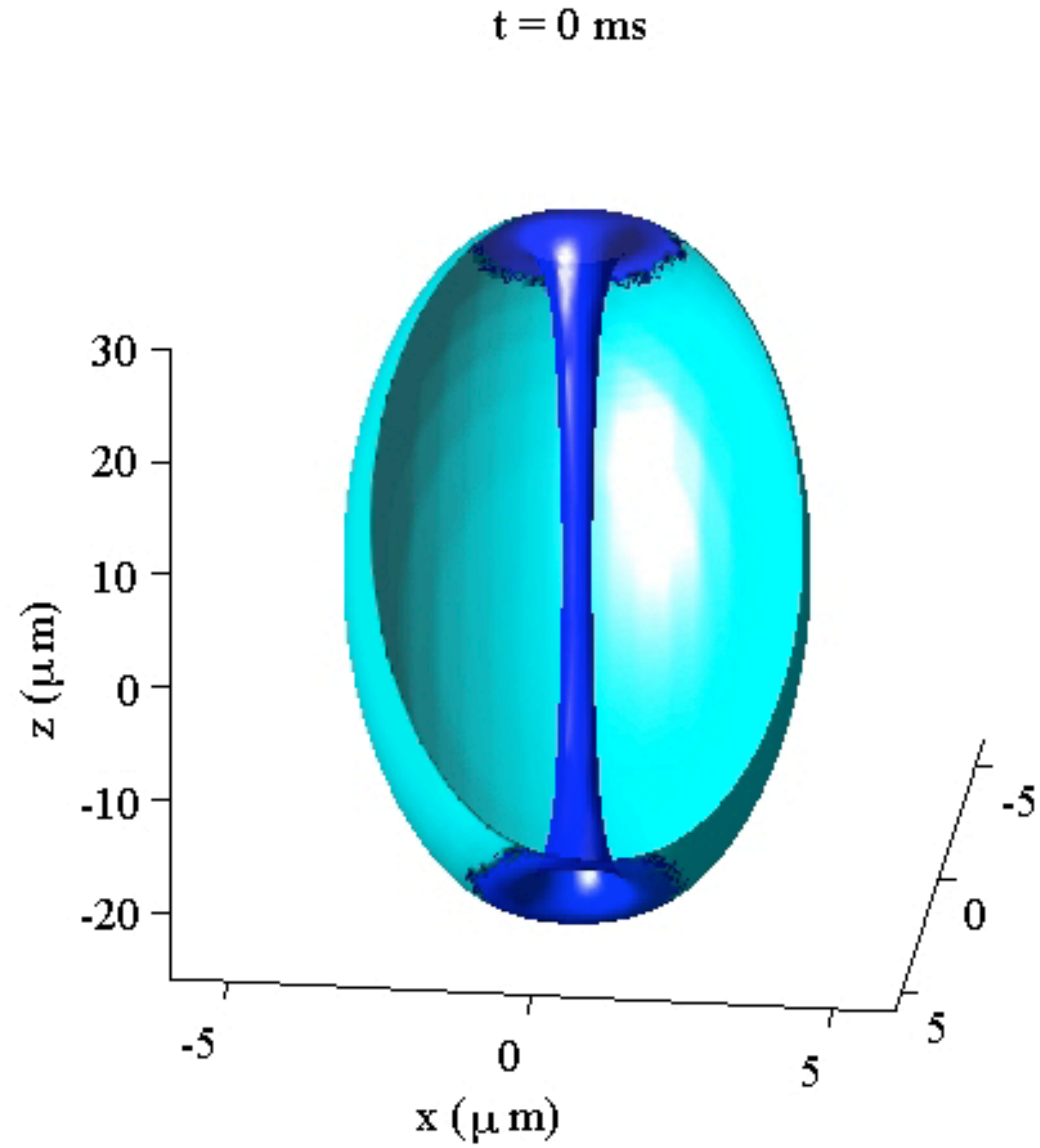




$$\omega(k) = 2\omega_\perp + \frac{\hbar k^2}{2m}$$

$$V_{\mathrm{pert}}(\mathbf{r}, t) = \epsilon m \omega_\perp^2 \cos(\Omega t) \cos(\mathbf{k}_z z)(x^2 + y^2)$$





T. P. Simula, T. Mizushima, and K. Machida, PRA (2008)

Tkachenko waves

VOLUME 91, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2003

Observation of Tkachenko Oscillations in Rapidly Rotating Bose-Einstein Condensates

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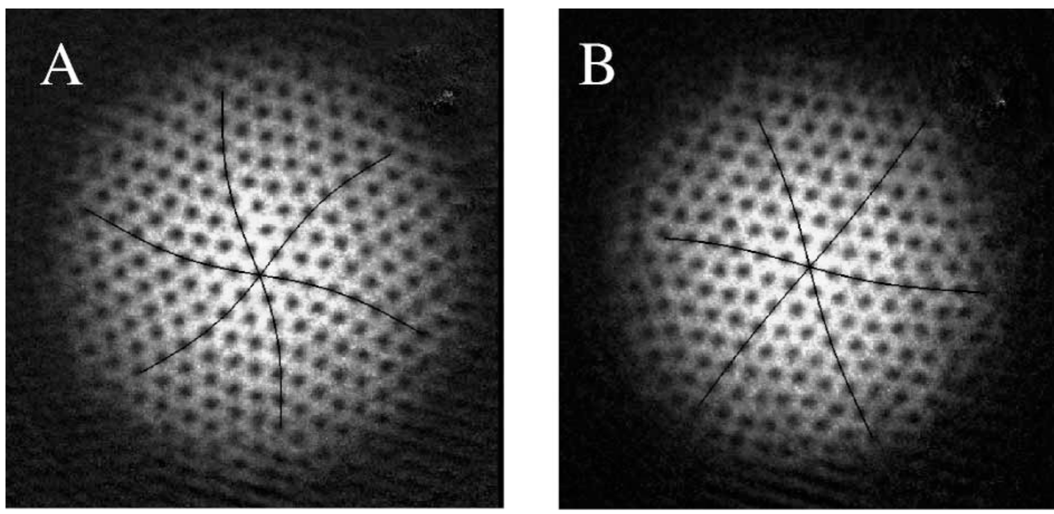
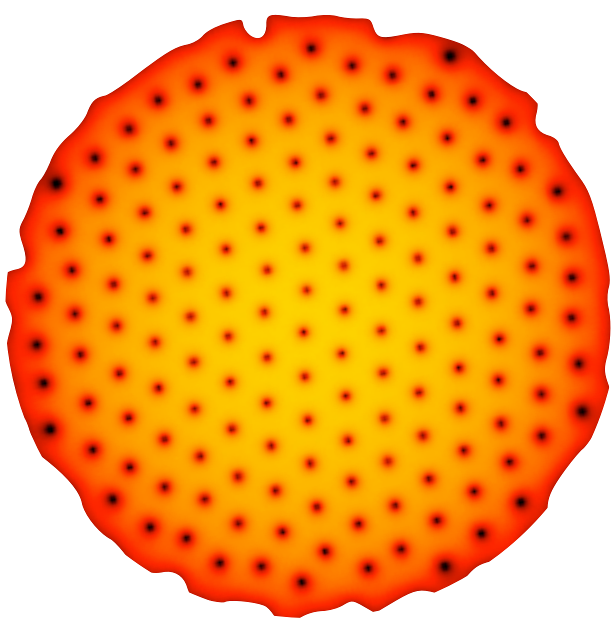
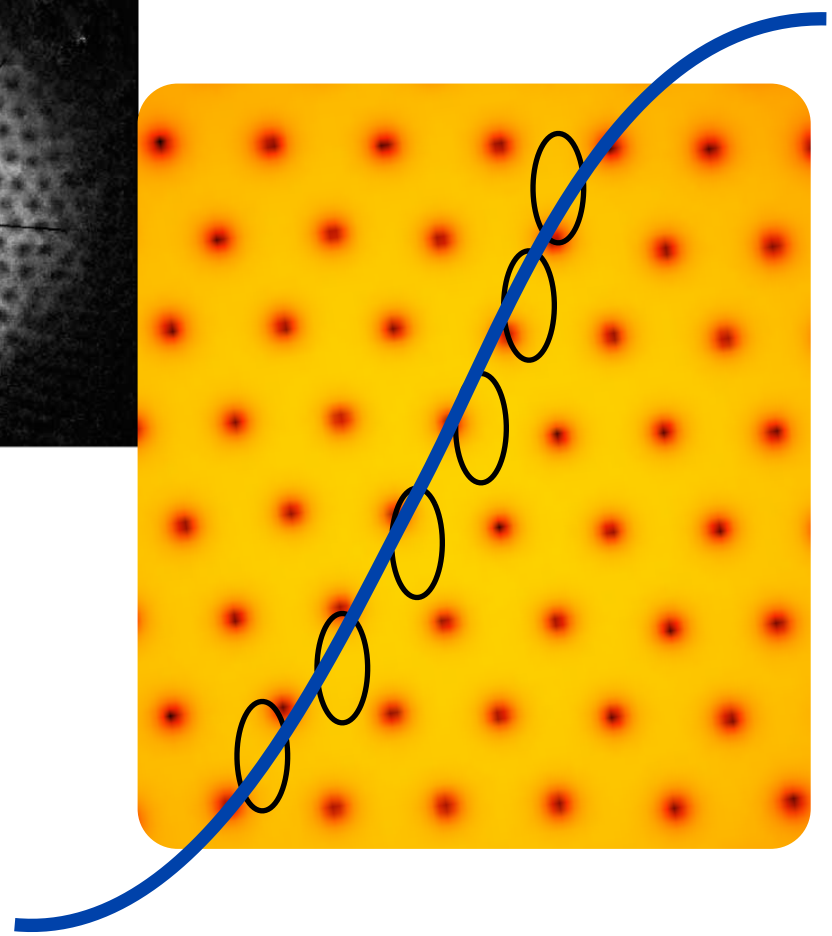
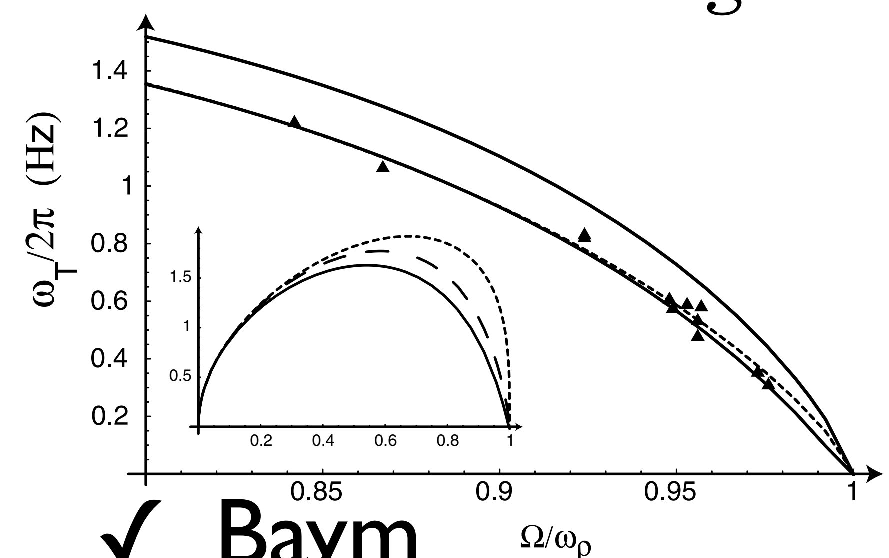
(Received 29 April 2003; published 5 September 2003)

$$\omega_T^2 = \frac{c_T^2 c_s^2 k^4}{4\Omega^2 + c_s^2 k^2}$$

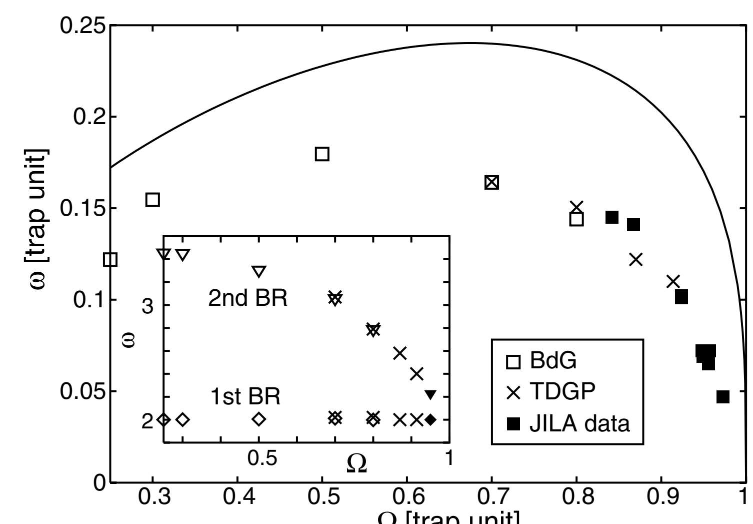






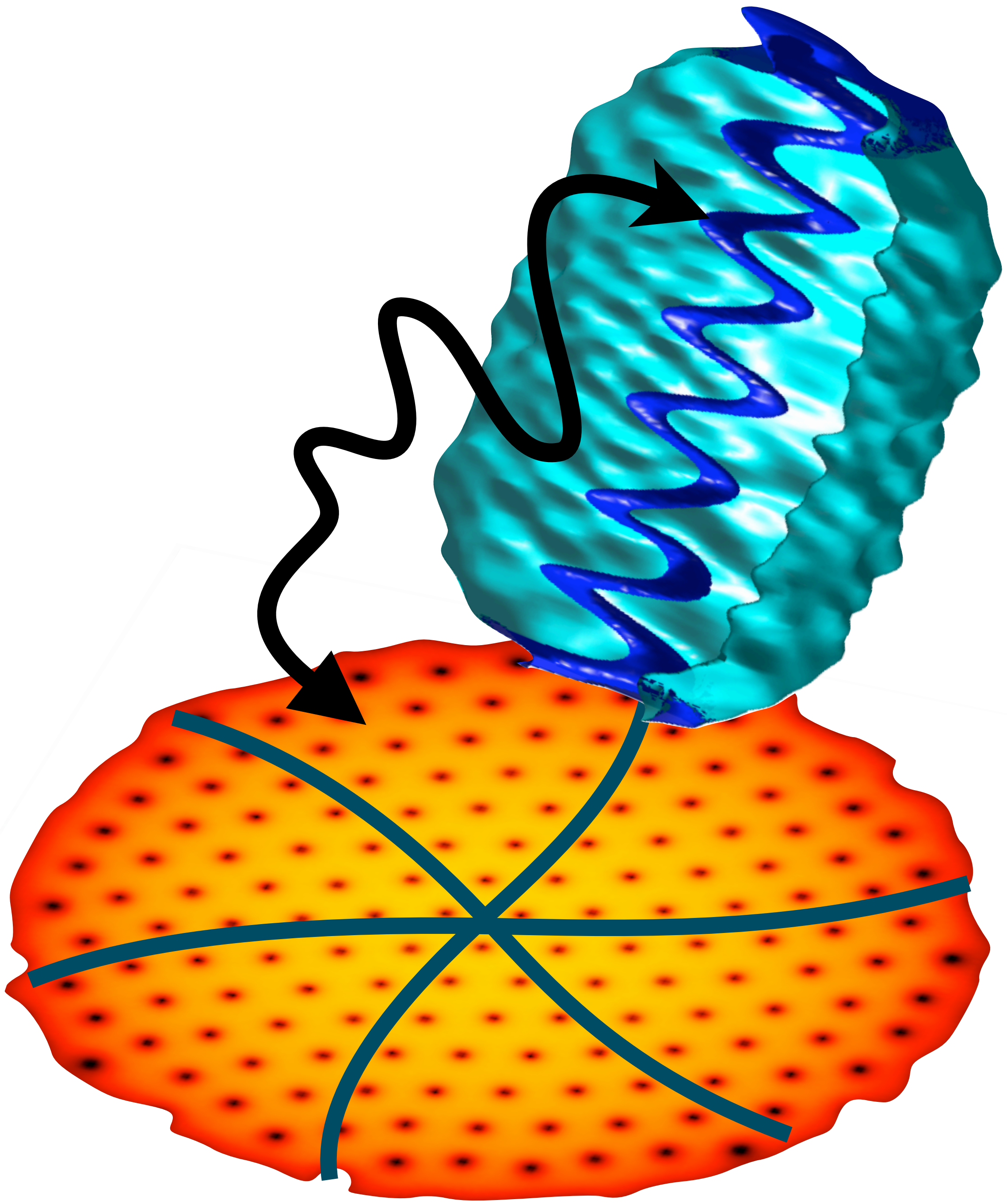


✓ Baym

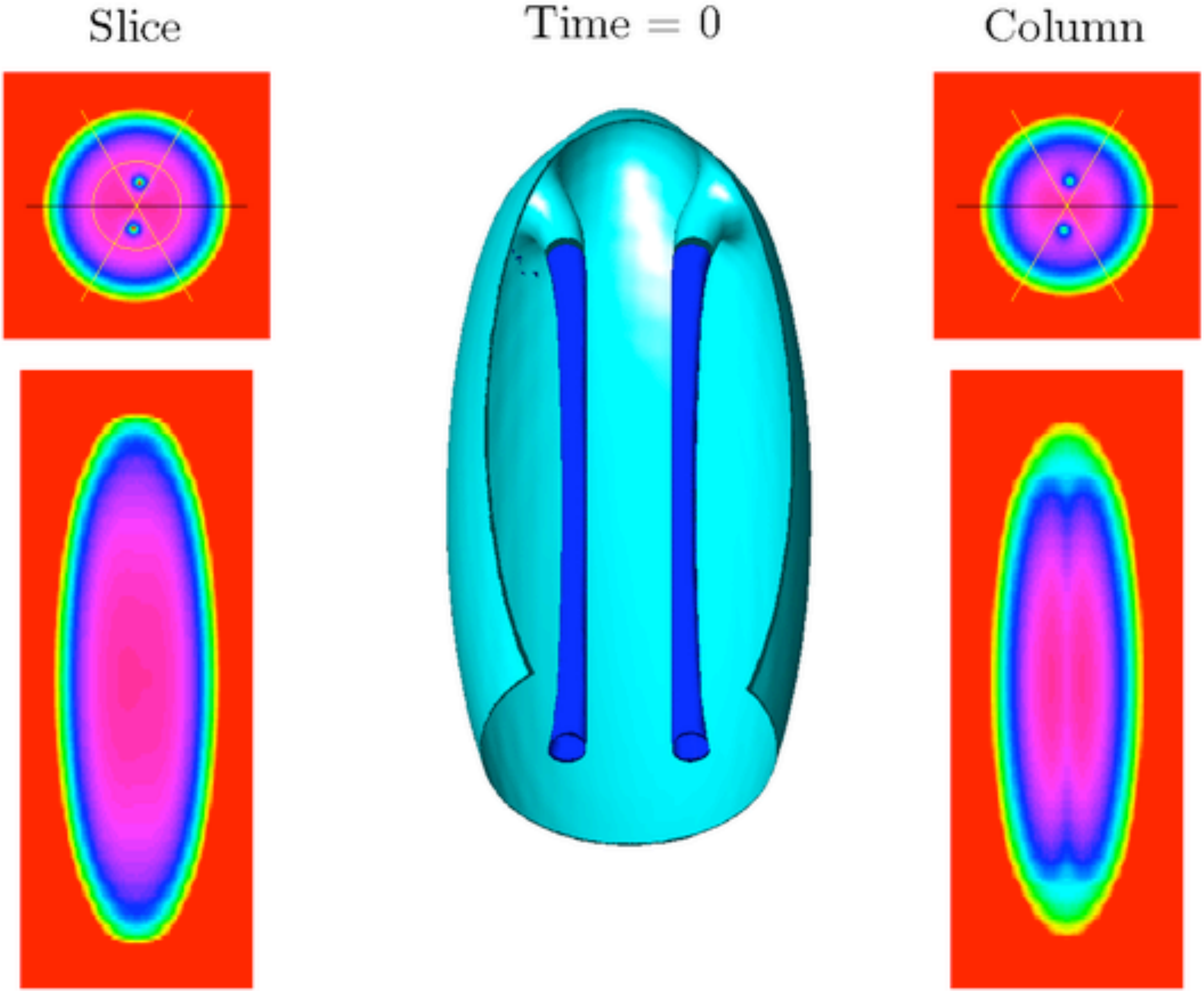


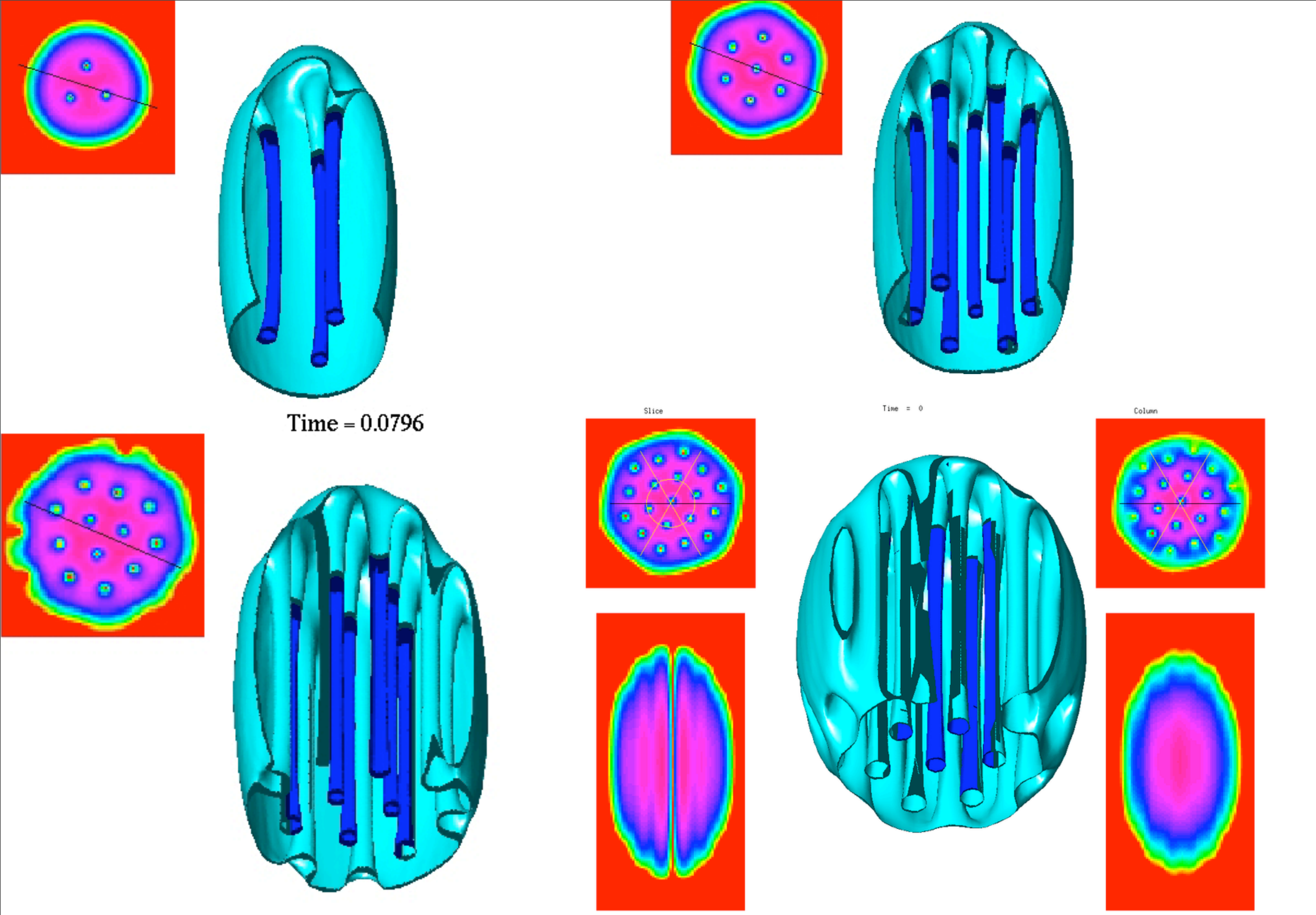
✓ Mizushima, et al.

Kelvin-Tkachenko modes

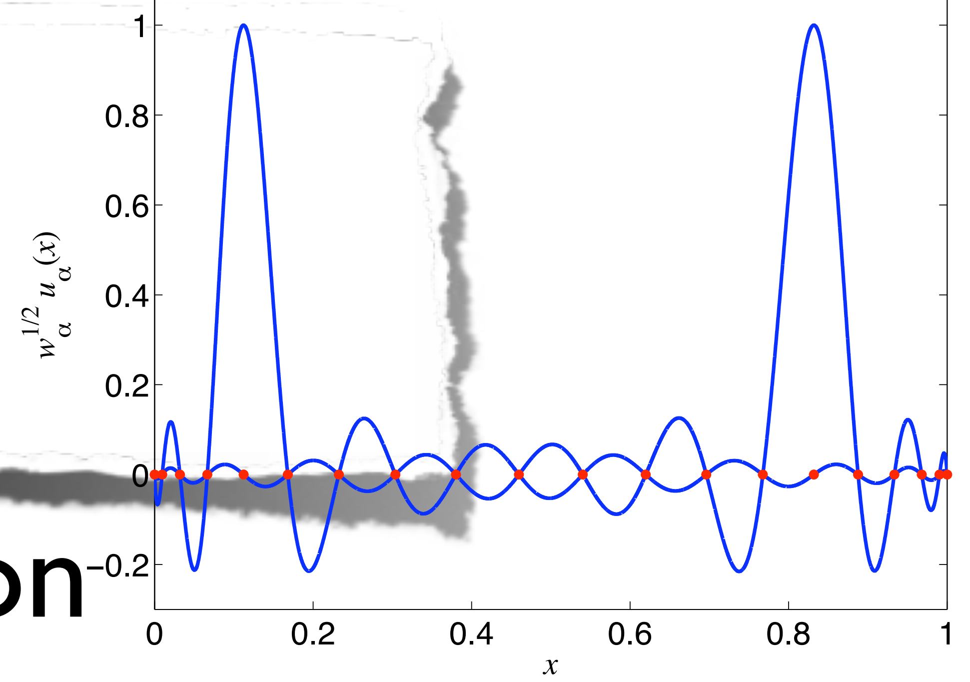


work in progress


Slice
Time = 0
Column


Time = 0.0796
Slice
Time = 0
Column

Methodology



✓ Discrete Variable Representation

- "massaged" polynomial basis (Hermite, Legendre, Laguerre...)

$$\{\phi_n, n = 0, \ldots, N-1\}$$

- quadrature rule

$$\langle f|g\rangle \equiv \int_a^b dx\, w(x) f(x) g(x) \approx \sum_{\alpha=1}^{N} w_\alpha f(x_\alpha) g(x_\alpha)$$

roots

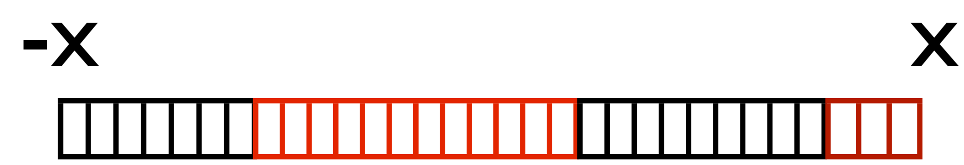
weights

- DVR basis functions $u_\alpha(x_\beta) = \dfrac{\delta_{\alpha\beta}}{\sqrt{w_\alpha}}$

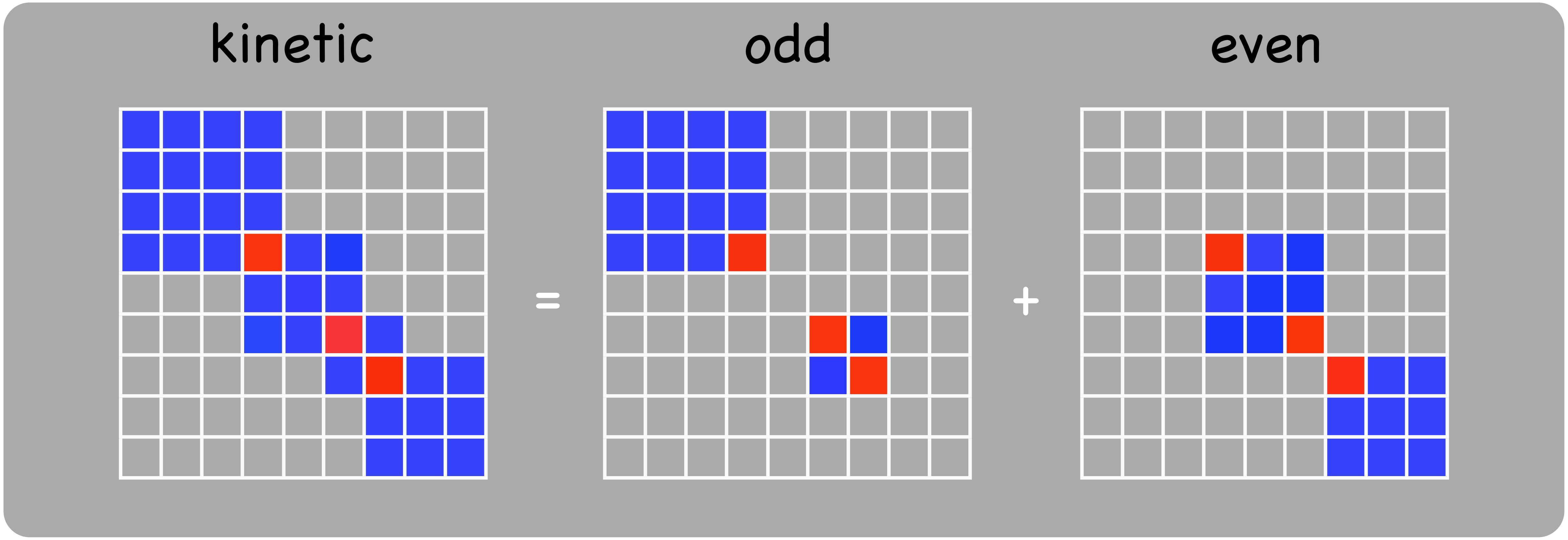
- diagonal potential operator

$$\langle u_\alpha|\hat{x}|u_\beta\rangle = \sum_q w_q u_\alpha^*(x_q) x_q u_\beta(x_q) = x_\alpha \delta_{\alpha\beta}$$

✓ finite-element extension



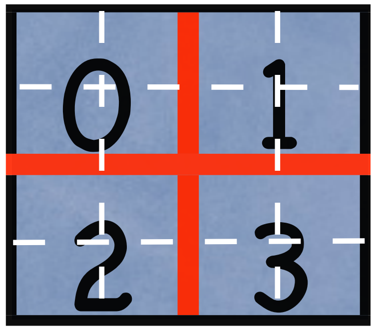
- very sparse matrix representation



- efficient temporal propagation (TDGPE)

$$e^{-iH\Delta t/\hbar} \approx e^{-iV\Delta t/2\hbar} e^{-iT\Delta t/\hbar} e^{-iV\Delta t/2\hbar}$$

- scalable parallelization using MPI



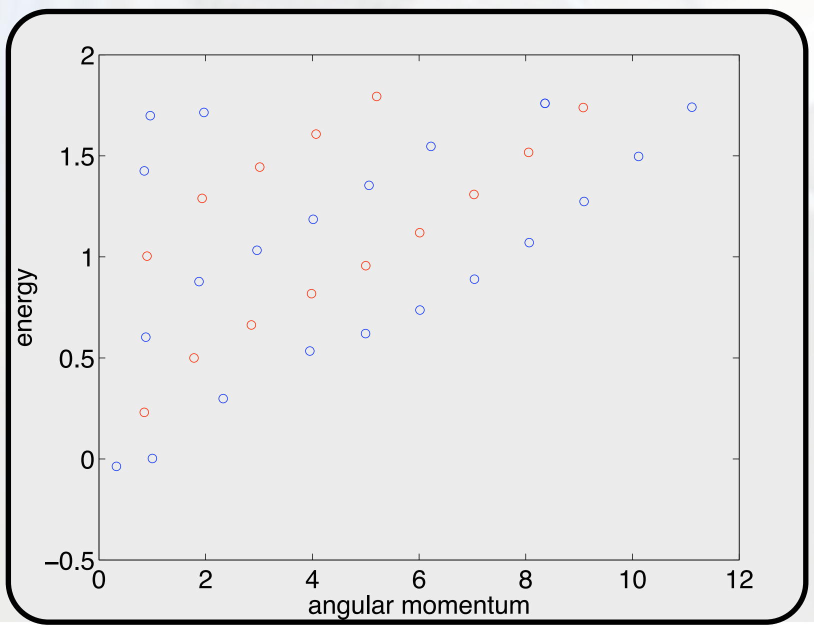
✓ parallelized diagonalization of large matrices

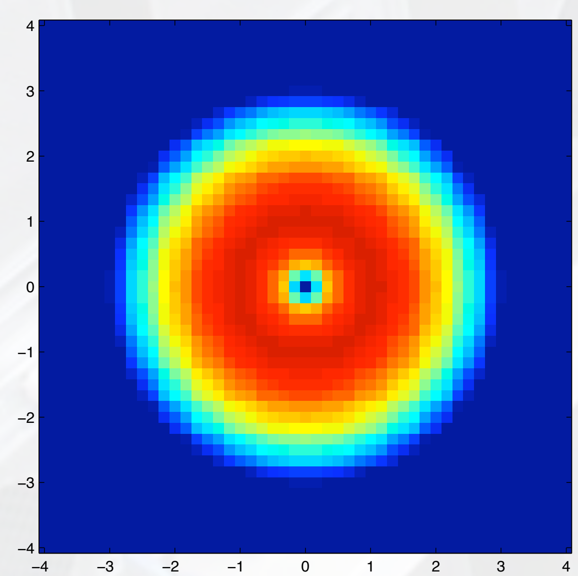
- Bogoliubov-de Gennes equations for Bose and Fermi systems

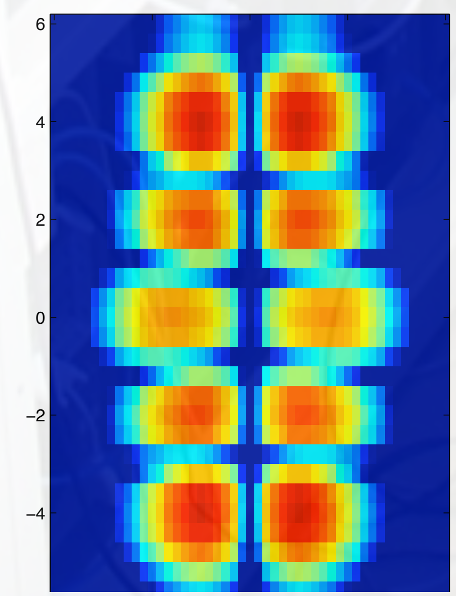
$$\begin{pmatrix} \mathcal{L} & \Delta \\ \pm\Delta^* & -\mathcal{L}^* \end{pmatrix} \begin{pmatrix} u_q \\ v_q \end{pmatrix} = E_q \begin{pmatrix} u_q \\ v_q \end{pmatrix}$$

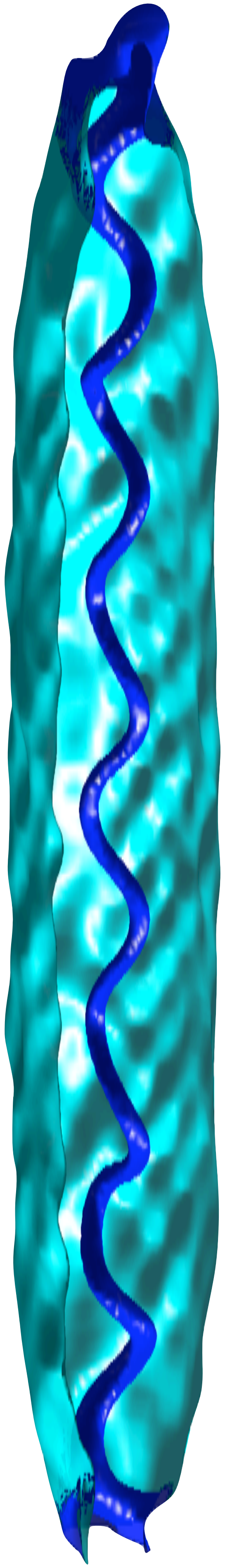
$$16 \text{ Byte} \times (2 \times 97 \times 97 \times 161)^2 \sim 133 \text{ TB}$$

- Arnoldi / Lanczos iteration in Krylov subspace

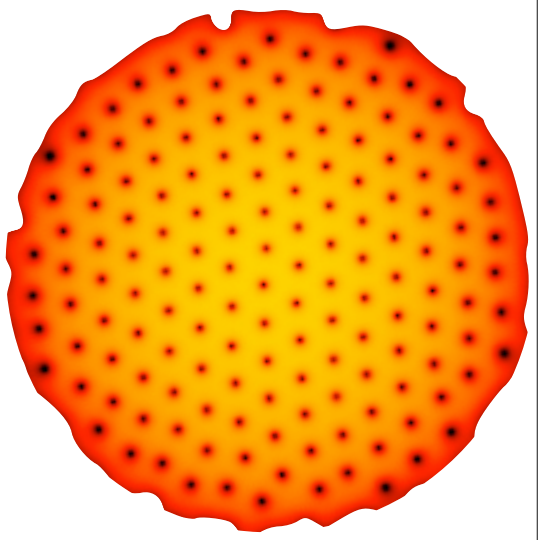








Feasible future directions



✓ *TDGPE + BdG in realistic* ***3D*** *systems*

- collective modes of rotating BEC ← in progress
- inclusion of dipolar interactions
- dynamics and collective excitations of F = 0,1,2,3... spinor condensates, turbulence...
- BdG studies of s,p,d...-wave paired Fermi-systems
- superfluidity of graphene