What we know and have yet to learn about spinor BEC?

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Contents

- Known knowns
- Known unknowns
- Unknown unknowns

Known Knowns
–Primer to Spinor BEC–

One Page Summary of Spin-1 BECs

T.Ohmi and K.Machida, J. Phys. Soc. Jpn. **67**, 1822 (1998)
T.-L.Ho, Phys. Rev. Lett. **81**, 742 (1998)

Bose symmetry requires that two particles collide either ↑↑ (a_2) or ↑↓(a_0). Not ↑←.

(a) $a_2/a_0<1$: ferromagnetic

spin-gauge symmetry
gauge rotation

$$\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix}=e^{i\phi}\sqrt{n}\hat{U}(\alpha,\beta,\gamma)\begin{pmatrix}1\\0\\0\end{pmatrix}=e^{i(\phi-\gamma)}\sqrt{n}\begin{pmatrix}e^{-i\alpha}\cos^2\frac{\beta}{2}\\ \frac{1}{\sqrt{2}}\sin\beta\\ e^{i\alpha}\sin^2\frac{\beta}{2}\end{pmatrix}$$

Continuous spin-gauge symmetry

coreless vortices
texture-induced supercurrent
circulation+Berry phase quantized.

(b) $a_2/a_0>1$: antiferromagnetic or polar

$$\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix}=e^{i\phi}\sqrt{n}\hat{U}(\alpha,\beta,\gamma)\begin{pmatrix}0\\1\\0\end{pmatrix}=e^{i\phi}\sqrt{\frac{n}{2}}\begin{pmatrix}-e^{-i\alpha}\sin\beta\\ \sqrt{2}\cos\beta\\ e^{i\alpha}\sin\beta\end{pmatrix}$$

Discrete spin-gauge symmetry

$\beta\to\pi-\beta$ (spatial inversion)
& $\phi\to\phi+\pi$ (gauge transformation).

fractional vortex
nematic order
knot excitation

Spin-2 BEC

$$2\otimes 2 = \underset{a_0}{0}\oplus \underset{a_2}{2}\oplus \underset{a_4}{4}\oplus \underbrace{1\oplus 3}_{\text{forbidden by Bose symmetry}}$$

Koashi & MU, Phys. Rev. Lett. **84**, 1066 (2000)
Ciobanu, et al., Phys. Rev. A **61**, 033607 (2000)
MU & Koashi., Phys. Rev. A **65**, 063602 (2002)

Interaction Hamiltonian

$$\hat{V} = \frac{1}{2}\int d\mathbf{r}\left[c_0 : \hat{n}^2 : + c_1 : \hat{\mathbf{F}}^2 : + c_2 \hat{S}^\dagger \hat{S}\right]$$

$$\hat{n} = \sum_m \hat{\psi}_m^\dagger \hat{\psi}_m \quad \cdots \text{ particle density}$$

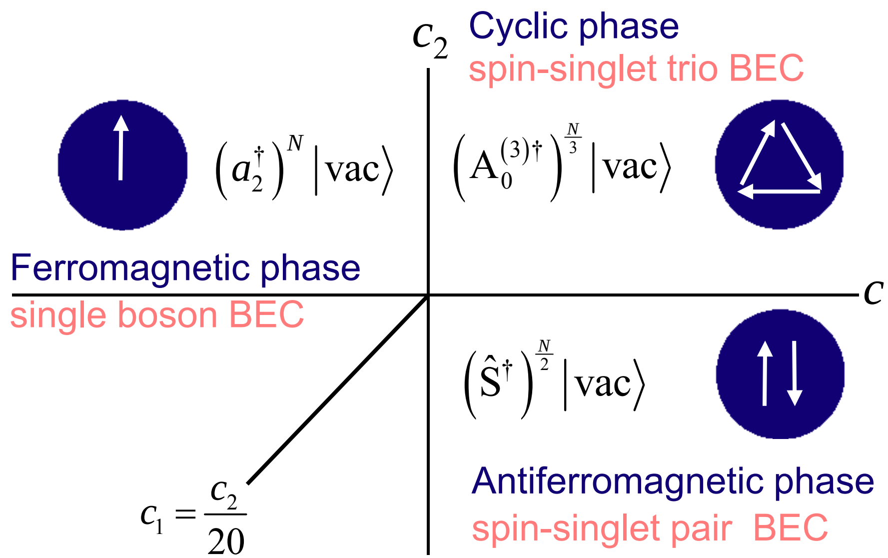
$$\hat{\mathbf{F}} = \sum_m \hat{\psi}_m^\dagger \mathbf{f}_{mn} \hat{\psi}_n \quad \cdots \text{ spin density}$$

$$\hat{S} = \sum_m \frac{(-1)^m}{\sqrt{5}} \hat{\psi}_m \hat{\psi}_{-m} \cdots \text{ spin-singlet pair amplitude}$$

$$c_0 = \frac{4\pi\hbar^2}{M}\frac{4a_2+3a_4}{7}$$

$$c_1 = \frac{4\pi\hbar^2}{M}\frac{a_4-a_2}{7}$$

$$c_2 = \frac{4\pi\hbar^2}{M}\frac{7a_0-10a_2+3a_4}{7}$$



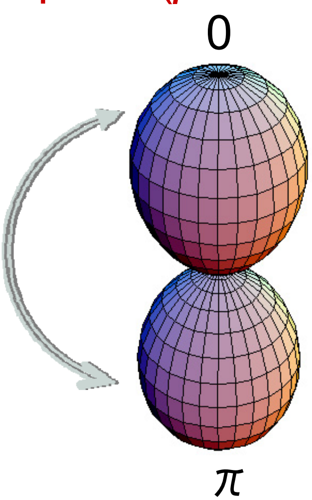
- Ground state of F=2 ^{87}Rb undecided!

(Tokyo-Gakushuinn problem)

- AFM phase divided into uniaxial and biaxial nematic phases; degeneracy lifted by zero-point fluctuations (Semenoff, et al.)

Fractional Vortices

F=1 polar (*pair singlet*) BEC



order parameter

$\psi \propto e^{i\phi}\cos\theta$

two-fold symmetry

The order parameter is invariant under

$\theta \to \pi - \theta$ (spatial inversion)

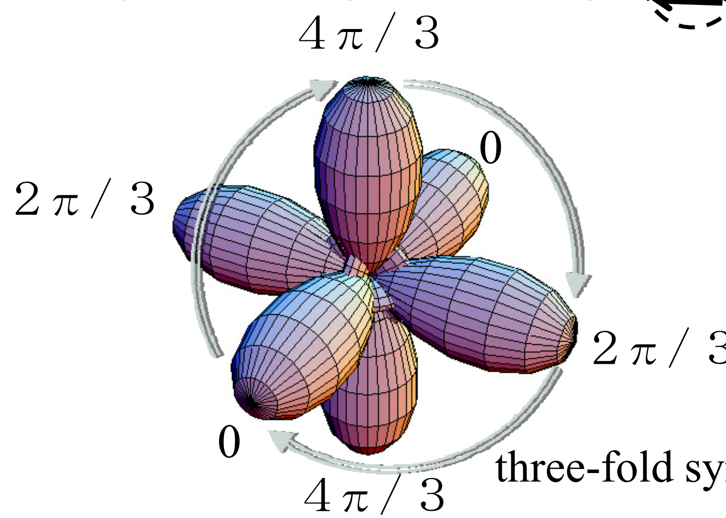
$\phi \to \phi + \pi$ (gauge transformation)

1/2 vortex

cf. no 1/2 vortex for *F=2* AF BEC

F. Zhou, Phys. Rev. Lett. **87**, 080401 (2001)

F=2 cyclic BEC (*trio singlet*)



three-fold symmetry

The order parameter is invariant under

$2\pi/3$ rotation about n=(1,1,1)

$\phi \to \phi + 2\pi/3$ (gauge transformation)

1/3 vortex

Y. Zhang, et. al. cond-mat/0404138
H. Mäkelä, J. Phys. A: Math Gen. **39**, 7423 (2006)
G. W. Semenoff and F. Zhou, Phys. Rev. Lett. **98**, 100401 (2007)

Some Issues on Spinor Vortices

- Core of a spinor BEC is generally not empty because the order parameter manifold ≠ $U(1)$.

- 1/2 vortex: the vortex core should be ferromagnetic.
  → FM vortex line in an AFM BEC= smoking gun of 1/2 vortex

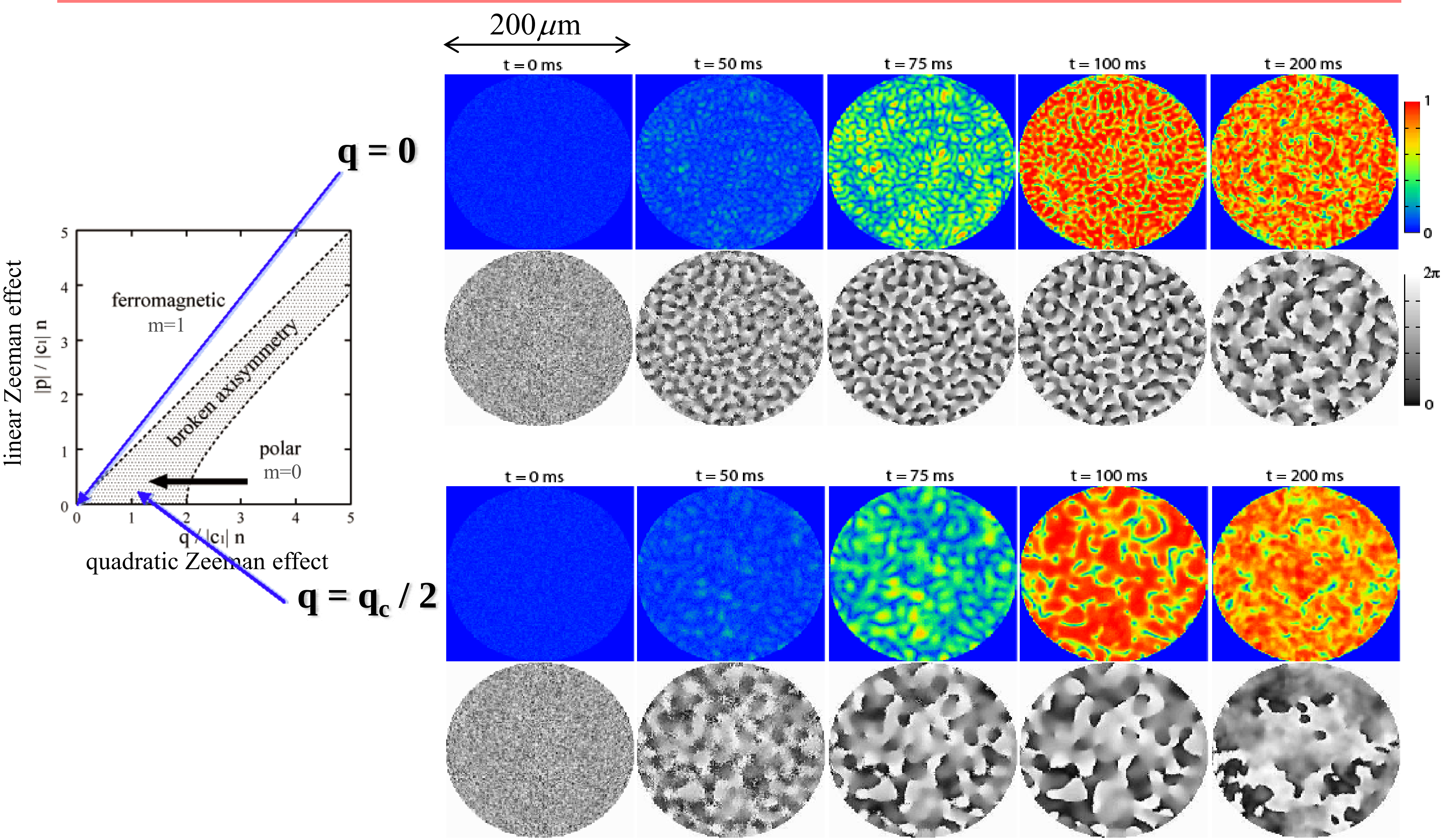
- 1/3 vortex: the core can be ferromagnetic or nematic.

- Order parameter manifold of the cyclic phase: non-Abelian.
  → Vortex reconnection, tangle, and turbulance
  highly nontrivial, open a new research field?
  (Kobayashi-san's talk tomorrow)

- More exotic phases, topological excitations etc., expected for higher spin states, such as in Cs BEC.

Known Unknowns
−Amplification of ultrasmall effects−

Spin Vortex Formation in a Quenched BEC



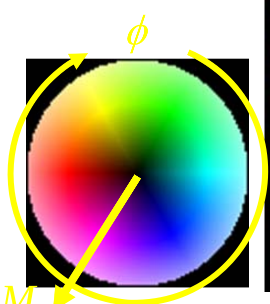
H. Saito, Y. Kawaguchi, and MU, PRA **76**, 043613 (2007)

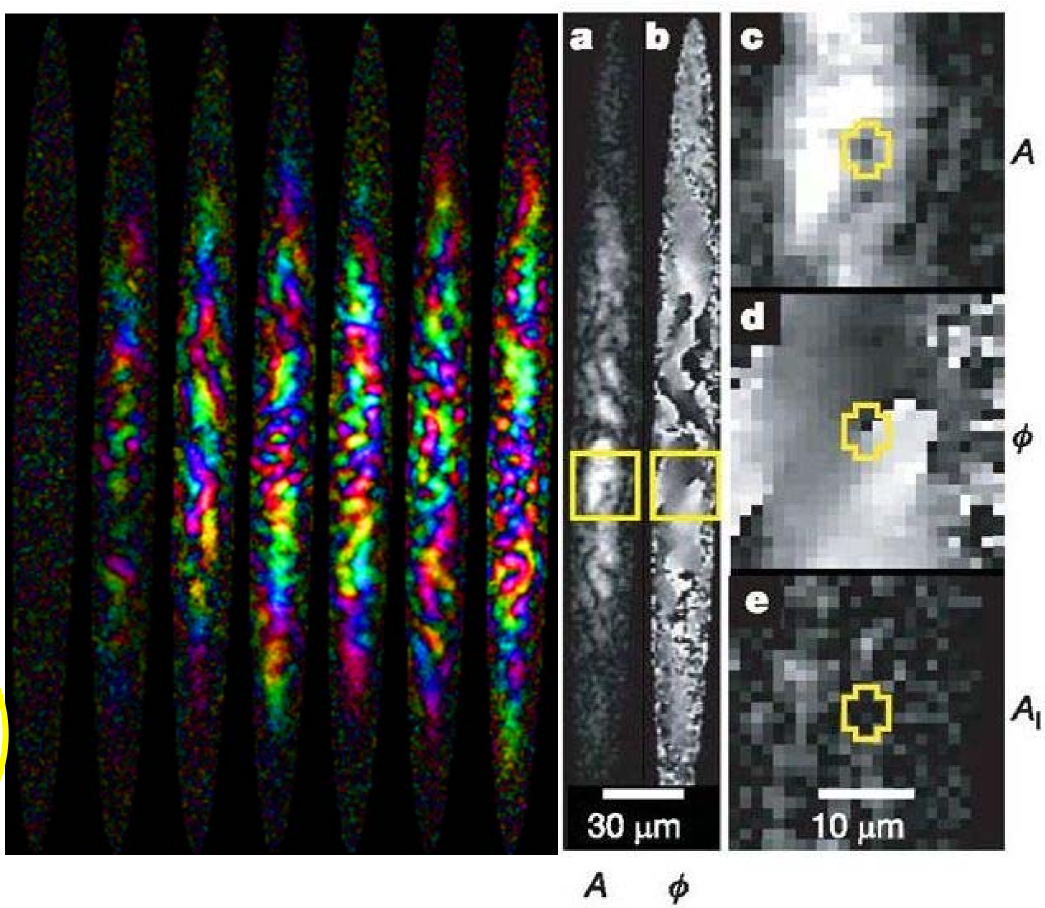
Spin Vortex observed by Berkeley Group

Sadler et al., Nature **443**, 312 (2006)

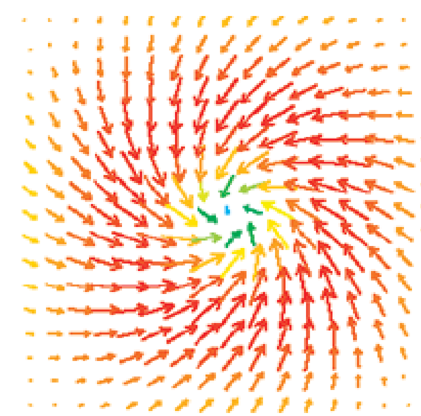
Snapshots of transverse magnetization of spin-1 ^{87}Rb BEC
initial condition: all atoms in the *m*=0 state

$$\begin{cases} \psi_1 \propto e^{\pm i\phi} \\ \psi_0 \propto 1 \\ \psi_{-1} \propto e^{\mp i\phi} \end{cases}$$





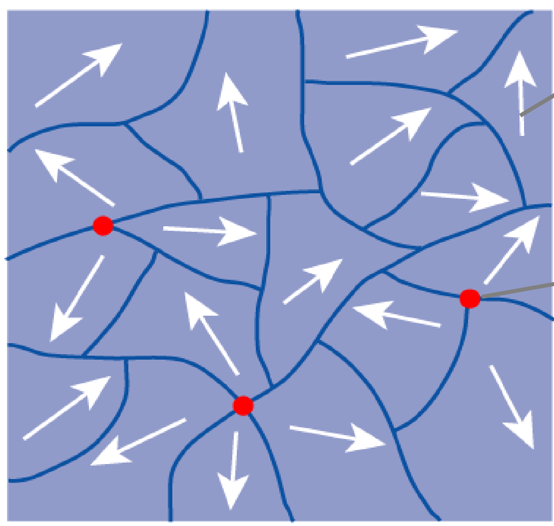
polar-core spin vortex
↕
chiral-symmetry broken state



H. Saito, Y. Kawaguchi, and M.U., PRL **96**, 065302 (2006)

Quench Experiments: A New Testing Ground for the Kibble Mechanism

Basic idea: spontaneous symmetry breaking at causally disconnected places generates topological defects where the order parameter causes frustration.

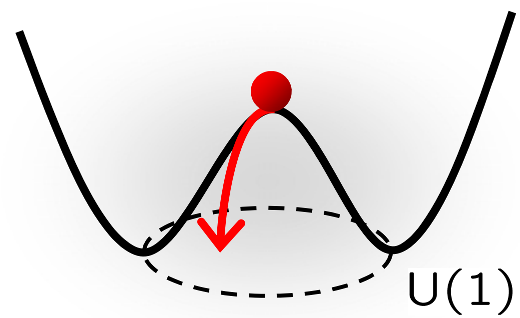


Order parameter

superfluid phase
magnetization

Topological defects

vortices



Cosmology, Superfluid He → **Cold Atoms**

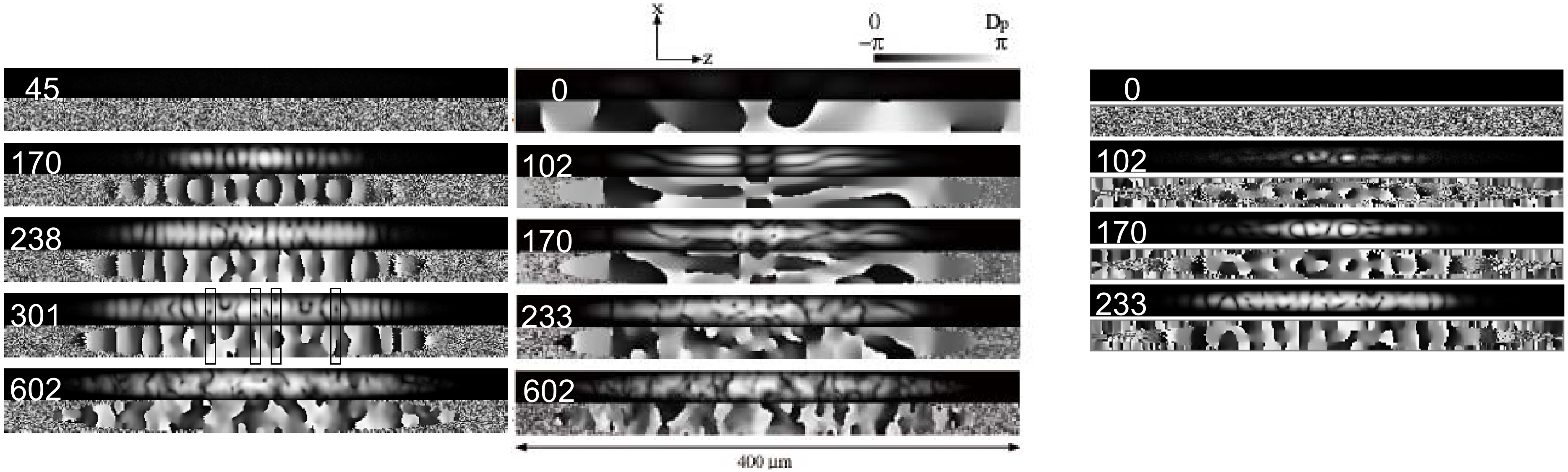
T. W. Kibble, J. Phys. A **9**, 1387 (1976)
W. H. Zurek, Nature **317**, 505 (1985)

ideal testing ground for the Kibble mechanism
a detailed quantitative comparison
between theory and experiments can be made.

Amplification of Initial Seeds due to Bosonic Stimulation

The pattern of transverse magnetization depends very sensitively on the initial seeds.

whie noise | colored noise | quantum fluctuations



Minute fluctuations (quantal, thermal, etc.) at the moment of quantum phase transition are amplified to yield observable magnetic structures.

→ Spin correlations give us information about the nature of initial seeds.

H. Saito, Y. Kawaguchi, and M.U., PRA **75**, 013602 (2007)

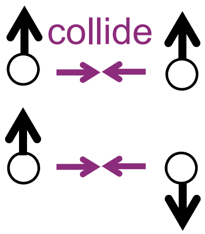
Known Unkowns
–Many-body effects–

Many-Body Gound State of a Spin-1 BEC

Law, et al., Phys. Rev. Lett. **81**, 5257 (1988)
Koashi and MU, Phys. Rev. Lett. **84**,1066 (2000)
Ho and Yip, Phys. Rev. Lett. **84**, 4031 (2000)

$$1 \otimes 1 = 2 \oplus 0 \oplus 1$$

forbidden by Bose symmetry



total spin 2: scattering length a_2

total spin 0: scattering length a_0

$a_2 < a_0$

$\uparrow\uparrow\uparrow \cdots\cdots \uparrow$ Bose ferromagnet $|\mathrm{BEC}\rangle = \frac{1}{\sqrt{N!}} \left(\hat{a}_1^\dagger\right)^N |\mathrm{vac}\rangle$

$a_2 > a_0$

Spin-singlet correlation $\uparrow\downarrow$ is favored: Bose antiferromagnet with no Neel order

$$\hat{S}^\dagger = \frac{1}{\sqrt{3}}\left(\hat{a}_0^{\dagger 2} - 2\hat{a}_1^\dagger \hat{a}_{-1}^\dagger\right) \qquad |\mathrm{BEC}\rangle \sim \left(\hat{S}^\dagger\right)^{\frac{N}{2}} |\mathrm{vac}\rangle \rightarrow \text{symmetry } \mathrm{U}(1) \qquad n_1 = n_0 = n_{-1} = \frac{N}{3}$$

cf. mean field: $\psi = \sum_{m=-1}^{1} a_m Y_1^m \cdots \frac{U(1) \times S^2}{Z_2}$ $\quad n_0 = 0$ → How can this discrepancy be reconciled ?

Connection between Many-Body Theory and Mean-Field Theory

In fact, mean-field theory breaks down at zero magnetic field, but its validity is quickly restored as the magnetic field increases.

Suppose that all bosons form spin-singlet pairs and all magnetic sublevels are equally populated.

$$|\mathrm{BEC}\rangle \sim \left(\hat{a}_0^{\dagger 2} - 2\hat{a}_1^\dagger \hat{a}_{-1}^\dagger\right)^{\frac{N}{2}} |\mathrm{vac}\rangle \;\rightarrow\; n_1 = n_0 = n_{-1} = \frac{N}{3}$$

As the magnetic field increases, singlet pairs are broken one by one via spin flip: ⇅ → ⇈.

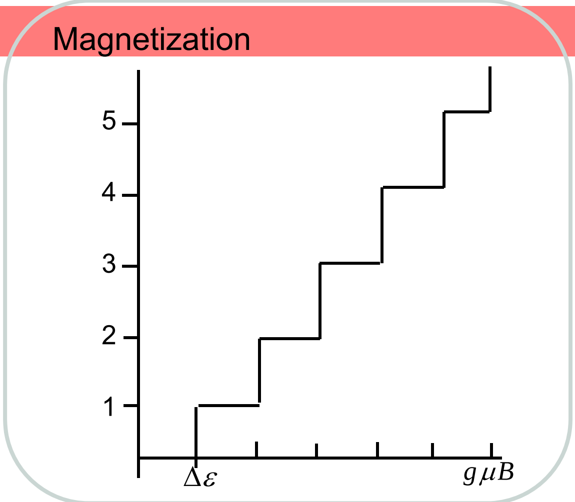
Connection between Many-Body Theory and Mean-Field Theory

When m pairs are broken, the many-body state becomes

$$\left(\hat{S}^{\dagger}\right)^{\frac{N}{2}}|\mathrm{vac}>$$

↓ spin flip of m singlet pairs

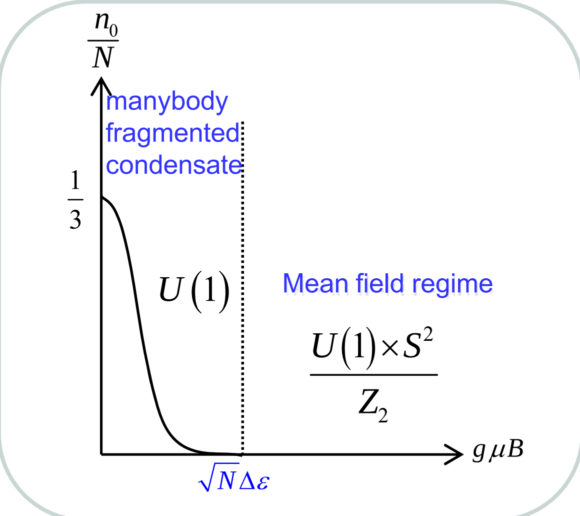
$$\left(\hat{a}_1^{\dagger}\right)^{2m}\left(\hat{S}^{\dagger}\right)^{\frac{N}{2}-m}|\mathrm{vac}\rangle \qquad \hat{S}^{\dagger}=\frac{1}{\sqrt{3}}\left(\hat{a}_0^{\dagger 2}-2\hat{a}_1^{\dagger}\hat{a}_{-1}^{\dagger}\right)$$



Each time a spin-singlet pair is broken, the m=0 component is exponentially suppressed due to *inverse* bosonic enhancement effect $\left(\hat{a}^{\dagger}|n\rangle=\sqrt{n+1}|n+1>\right)$.

The m =0 component virtually disappears at $m \sim N^{1/2}$, beyond which mean field theory is restored.

➢The fractional dependence on *N* indicates that manybody effects appear in the mesoscopic regime.
➢Relevant magnetic field ~ 10μG



M. Koashi & MU, Phys. Rev. Lett. **84** 1066 (2000)

“ Meissner Effect ” of the Antiferromagnetic Spin-2 BEC

Minimize the spin-dependent part of the Hamiltonian

# of trio singlets

$$\hat{H}^{\text{spin}} = \frac{c_1}{2V^{\text{eff}}} :\hat{\mathbf{F}}^2: + \frac{2c_2}{5V^{\text{eff}}} \hat{S}^\dagger \hat{S} - \underbrace{p\hat{F}_z}_{\text{Zeeman term}} \qquad p = g\mu B \qquad |N, N_s, F, F_z; \lambda\rangle$$

# of spin-single pairs

with respect to magnetization F_z :

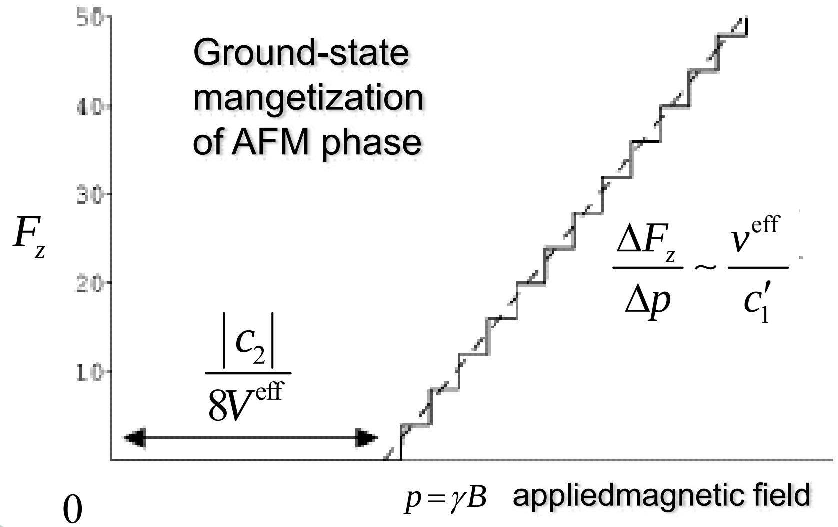
This term counteracts B because $c_2 < 0$.

$$E(F_z, l) = \frac{c_1'}{2V^{\text{eff}}} \left[F_z - \frac{g\mu V^{\text{eff}}}{c_1'} \left(B + \frac{c_2}{8g\mu V^{\text{eff}}} \right) + \frac{1}{2} \right]^2 - \frac{c_2}{40V^{\text{eff}}} l(l + 2F + 6) + \text{const.}$$

$$l = 2(N - 2N_s) - F_z$$

$$c_1' \equiv \frac{1}{V^{\text{eff}}}\left(c_1 - \frac{c_2}{20}\right) > 0$$

$$c_2 < 0 \quad \text{for AFM}$$



Physically, the $F = 2$ spin-singlet pair condensate acts to screen the external magnetic field until the magnetic field reaches a critical value.

↓

$B^{\text{critical}} \sim 1/V^{\text{eff}}$

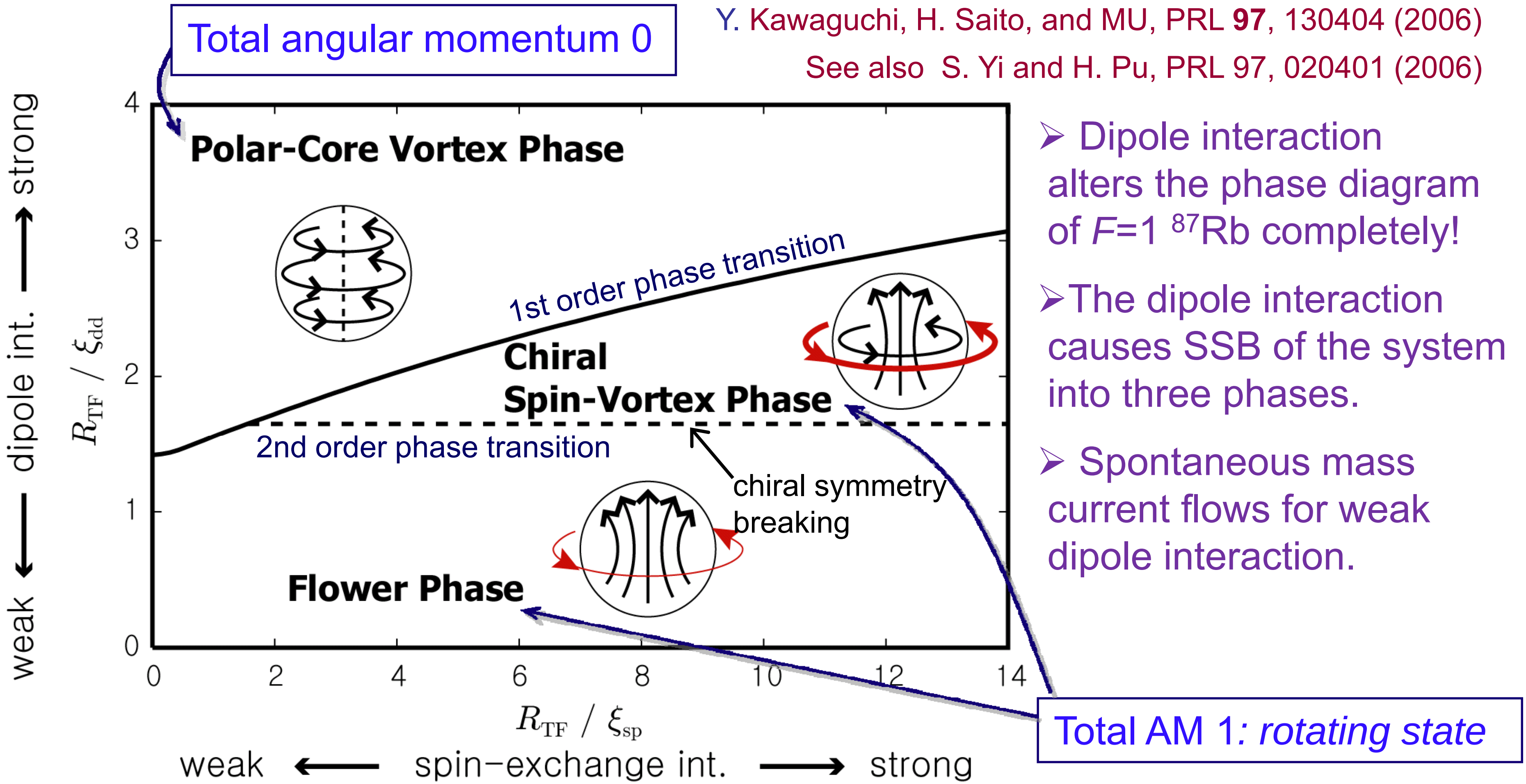
mesoscopic Meissner effect

MU & M. Koashi, Phys. Rev. A **65**, 063602 (2002)

Phase Diagram of F=1 ^{87}Rb BEC

Y. Kawaguchi, H. Saito, and MU, PRL **97**, 130404 (2006)

See also S. Yi and H. Pu, PRL 97, 020401 (2006)



- Dipole interaction alters the phase diagram of *F*=1 ^{87}Rb completely!
- The dipole interaction causes SSB of the system into three phases.
- Spontaneous mass current flows for weak dipole interaction.

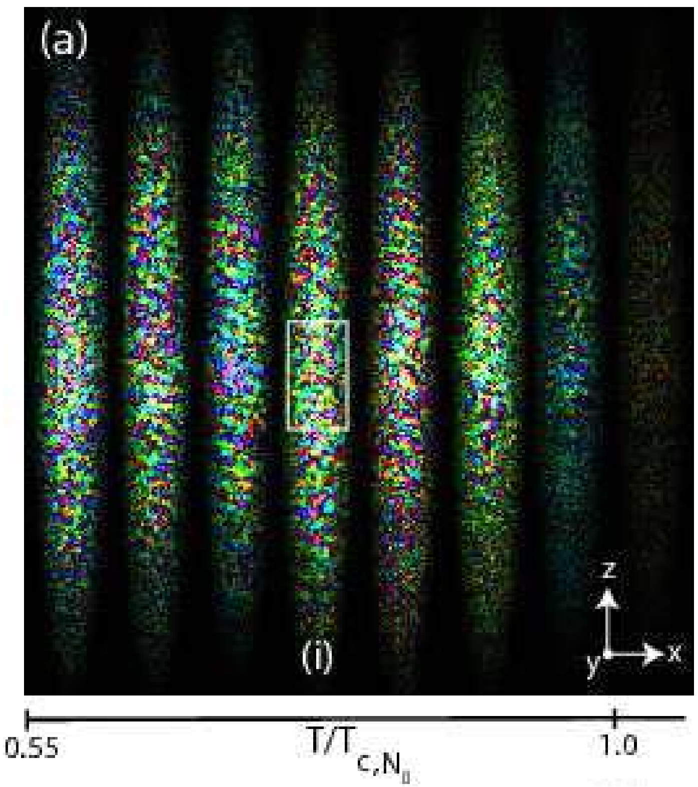
Yet another untapped research arena:
spinor-dipolar BEC at ultralow magnetic field + manybody effects?

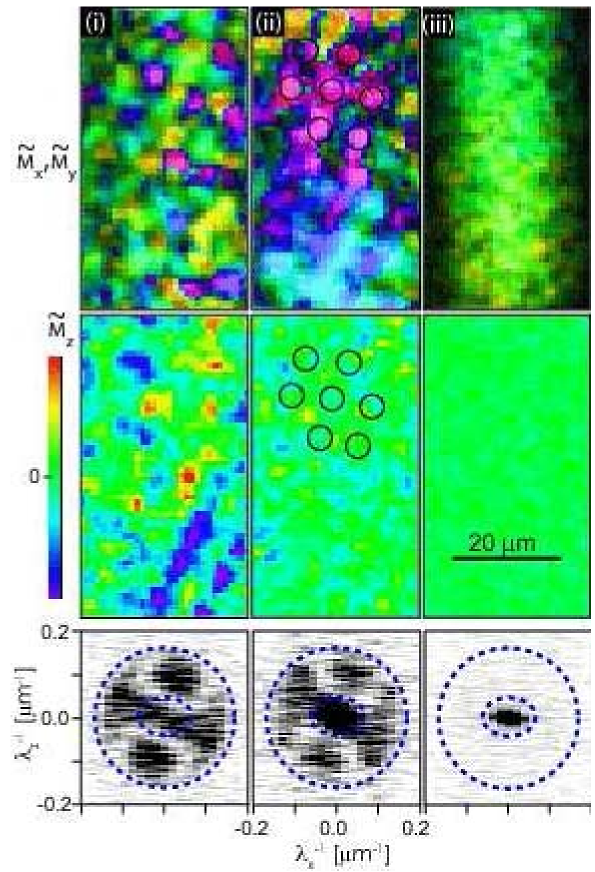
Prediction: Lower the B-field down to 10 μG; then BEC will rotate spontaneously.

–Crystalline Magnetic Order–

Magnetic Crystallization of F=1 ^{87}Rb

M. Vengalattore, et al. arXiv:0901.3800





Summary

- spinor-dipolar BEC: a cornucopic of symmetry breaking-

Quantum phases, Magnetism, Topology

- New quantum phases: cyclic phase, chiral spin-vortex phase
- Fractional vortices: spin-1 polar BEC: 1/2 vortex
  spin-2 cyclic BEC: 1/3 vortex

  What's the structure of the vortex core?

- Topological defects: chiral spin vortex
  ideal testing ground for Kibble mechanism
- Amplification of ultra-weak effects
  initial seeds for nucleation (quantum, thermal, kinetics)
  amplification of fundamental symmetry breaking: parity, *etc.*
- Crystalline Magnetic Order