Some Aspects of Quasi-classical Theory
- Boundary Conditions and Application to Dynamical Problems -

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Quasi-classical Green's function is a useful tool to describe the inhomogeneous states in Fermi superfluids and superconductors. In *s*-wave superconductors, inhomogeneity is induced, for example, by proximity effect or by magnetic fields. In anisotropic pairing states, the existence of the boundary itself induces inhomogeneous states and the pairing state near the boundary is significantly modified from that in the bulk.

In this talk, I first briefly review the conventional derivation of the Eilenberger equation of the quasi-classical Green's function from the Gor'kov equation, which is called "left-right trick". In the left-right trick, important informations carried by the Gor'kov Green's function, the normalization and the boundary condition, are lost. One usually has to supplement the so called "normalization condition" by hand. No information on the boundary condition is given by the Eilenberger equation.

I shall talk on another derivation which has been developed by our group. The basic idea is to go back to the Gor'kov equation and to write the Gor'kov Green's function as a product of the rapidly oscillating factor with the Fermi wave length and the slowly changing factor with the coherence length. We can show that the slowly varying factor is a counterpart to the quasi-classical Green's function. Using the information that the Gor'kov Green's function has, we can obtain the formal solution for the counterpart that has already satisfied the boundary condition. In finite size systems, the counterpart still has rapidly oscillating part, which comes from the multiple scattering within the finite system. After averaging the counterpart over that rapid oscillation, we obtain the quasi-classical Green's function that satisfies the normalization condition.

Recently, Riccati representation of the quasi-classical Green's function has been proposed, which has a form convenient for numerical computation. We can show that this representation can be also naturally obtained by our prescription.

Our method can be applied also to the Keldysh quasi-classical Green's function which is one of the tools to treat non-equilibrium systems. We have applied it to the transverse acoustic impedance of superfluid ^{3}He and obtained a result in good agreement with the recent experiments by Tokyo Tech group. I will show some new results on the transverse acoustic impedance.