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Boundary and Impurity Effects on Anisotropic Superfluids and Superconductors

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共同研究者

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1. Transverse Acoustic Impedance of Superfluid $^3\text{He-B}$
2. Sound Propagation in Liquid ^3He in Aerogel
3. Electronic Raman Scattering of Sr_2RuO_4
4. Future Problems

Transverse Acoustic Impedance of Superfluid $^3\text{He-B}$

Acoustic Impedance

$$Z = \Pi_{xz}/v$$

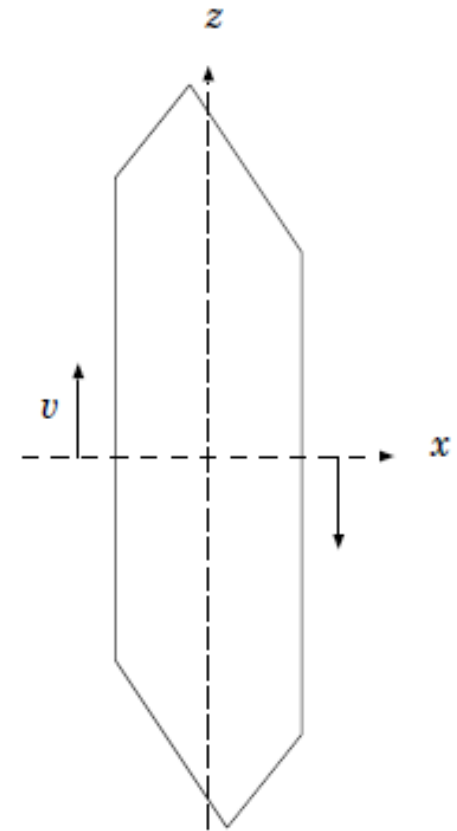
Π_{xz} : Stress Tensor of Liquid

Resonance Frequency of Quartz

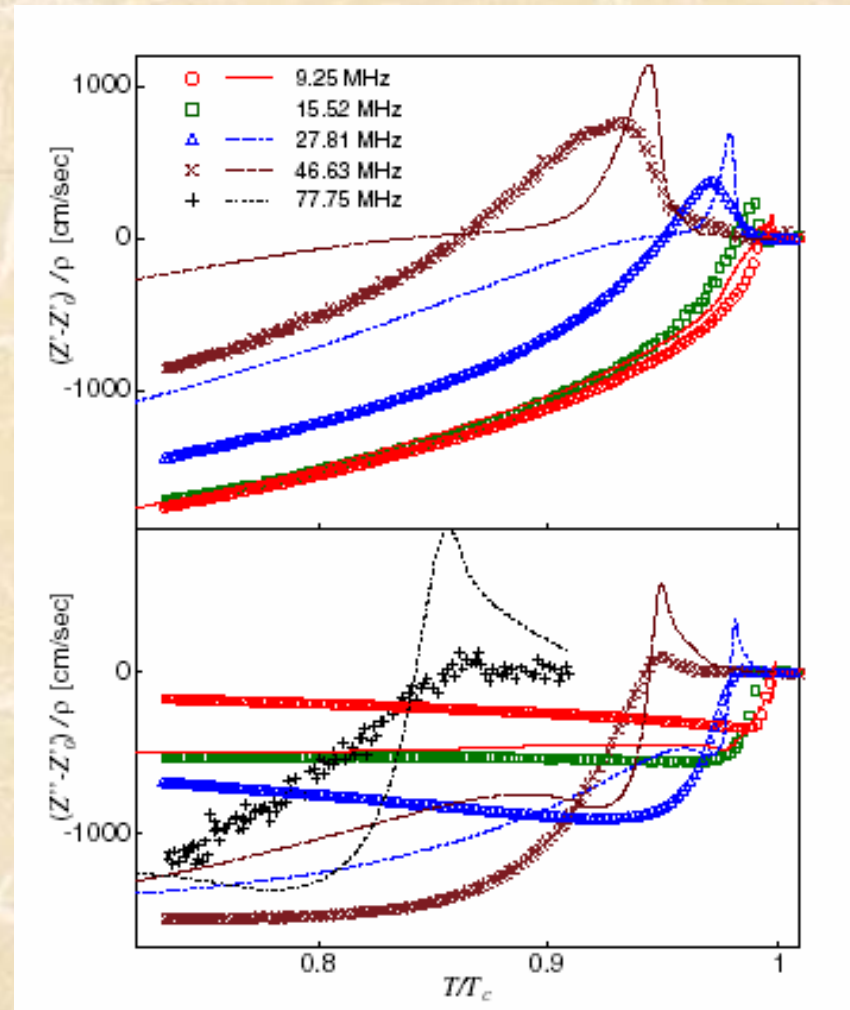
$$Z' = \frac{\pi}{4} Z_Q \frac{1}{Q}$$

$$Z'' = \frac{\pi}{2} Z_Q \delta\omega$$

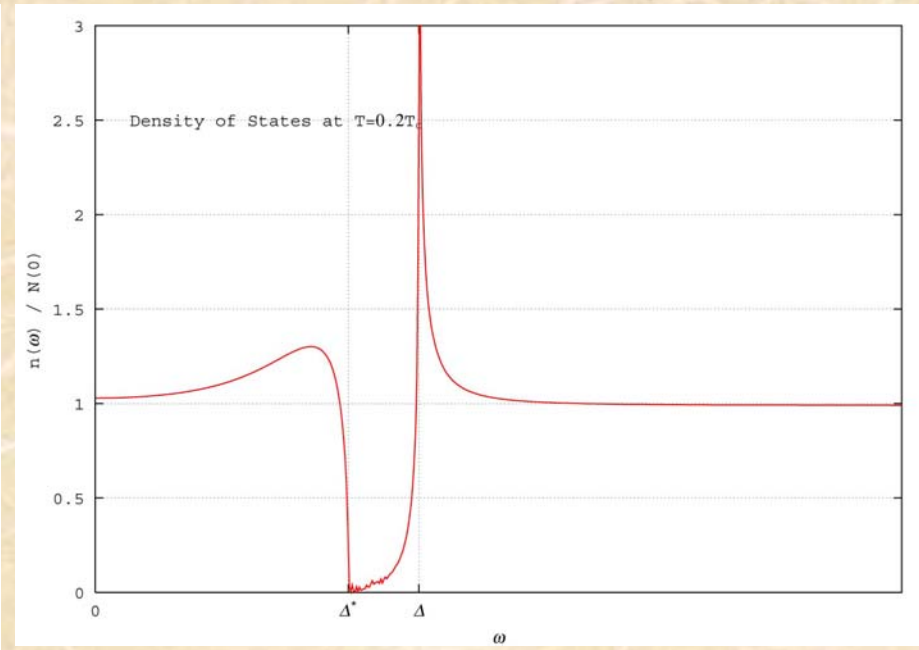
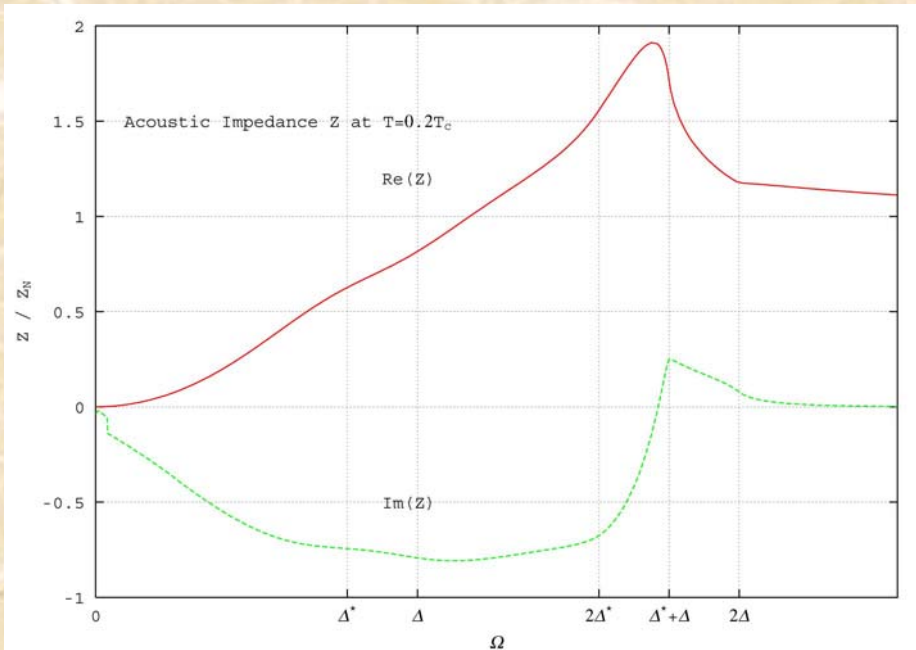
$$Z_Q = \rho_Q c_t$$



Aoki et al., Phys. Rev. Letters **95**, 075301 (2005)



Frequency Dependence of $Z=Z'+iZ''=P_{xz}/u_{\text{wall}}$



Structure in the acoustic impedance can be related to the pair excitations of midgap quasi-particle and propagating quasi-particle.

Acoustic Surface Spectroscopy

Random S-matrix Model

Nagato et al. (1996)

$$S = -\frac{1 - i\eta}{1 + i\eta} \quad (\eta_{KQ}: \text{Hermite matrix})$$

η : Random variable to describe the surface roughness

$$\overline{\eta_{KQ}} = 0$$

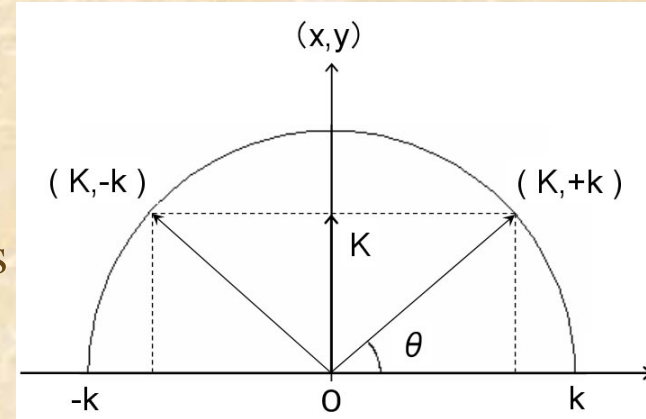
$$\overline{\eta_{KQ}^* \eta_{K'Q'}} = \eta^{(2)}(K - Q) \delta_{K-Q, K'-Q'}$$

$$\eta^{(2)}(K - Q) = \frac{2W}{\sum_Q 1}$$

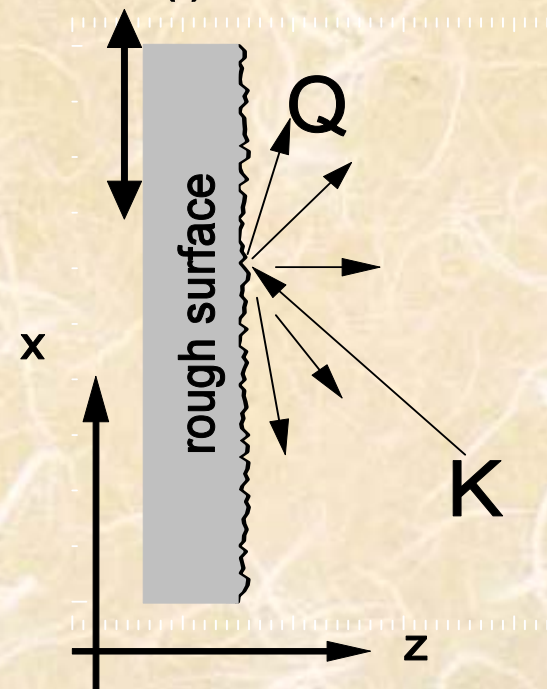
W : Parameter to describe the roughness of the surface

$$W = 0 \quad \longleftrightarrow \quad W = 1$$

Specular \longleftrightarrow Diffusive

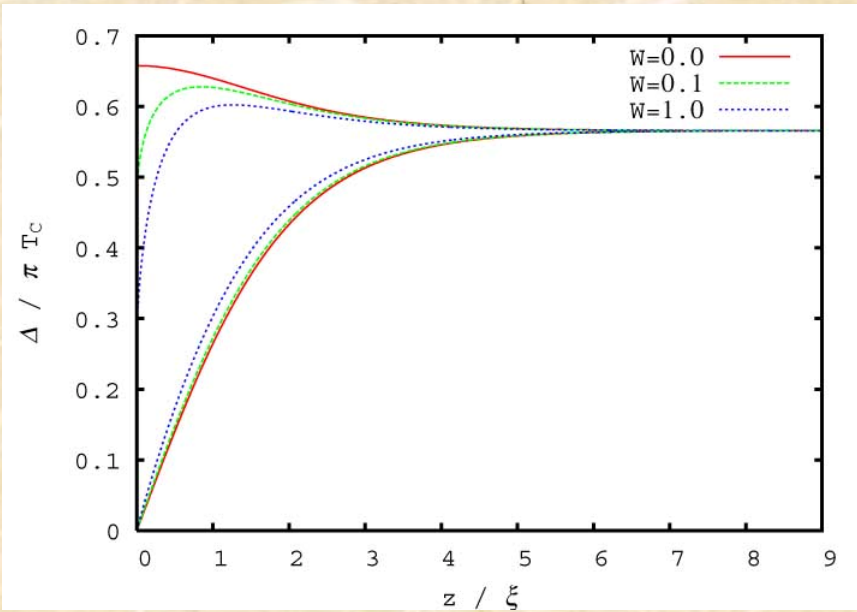


$$u = dR(t) / dt$$

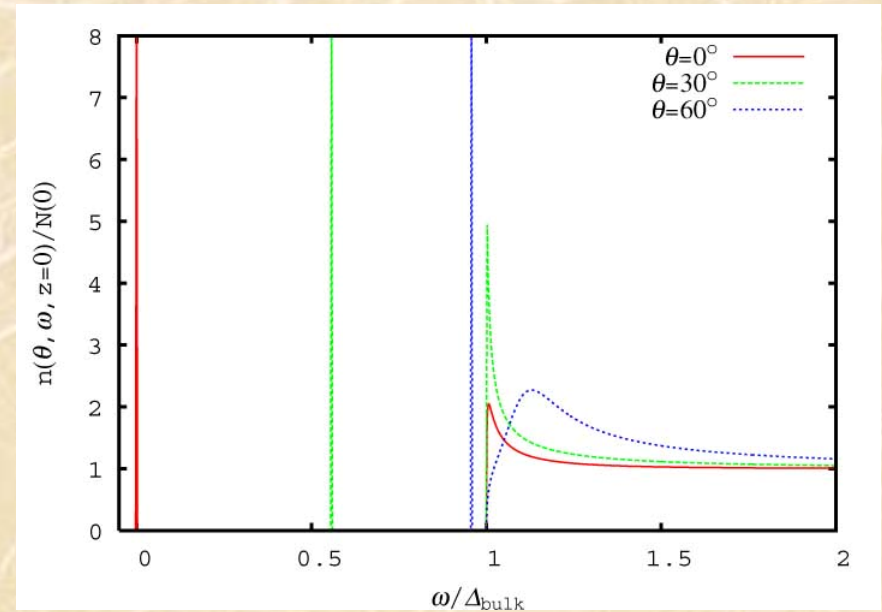


Surface Effects on BW State

Orderparameter

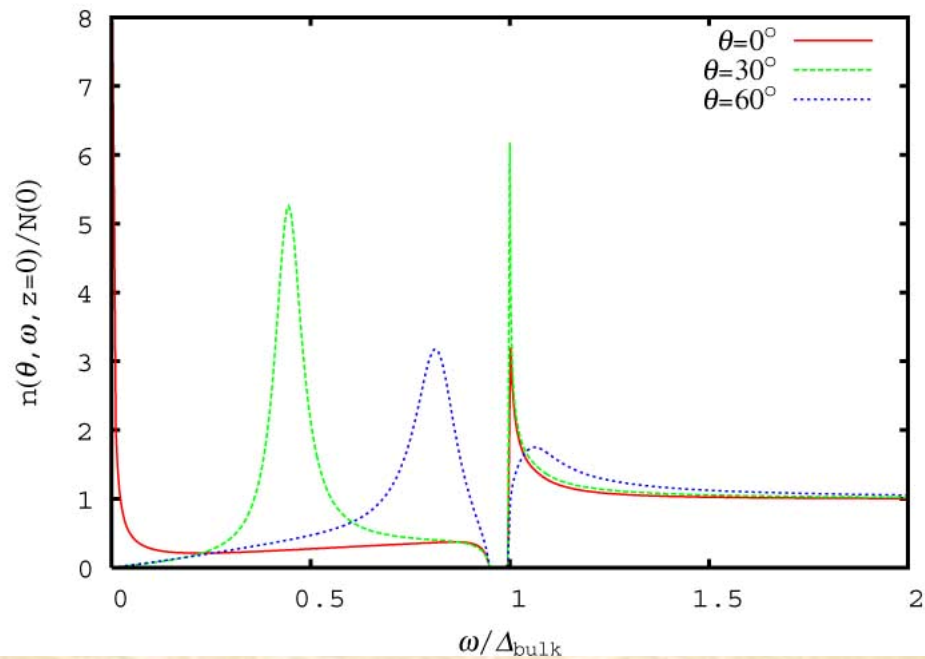


Angle Resolved Density of States at Surface

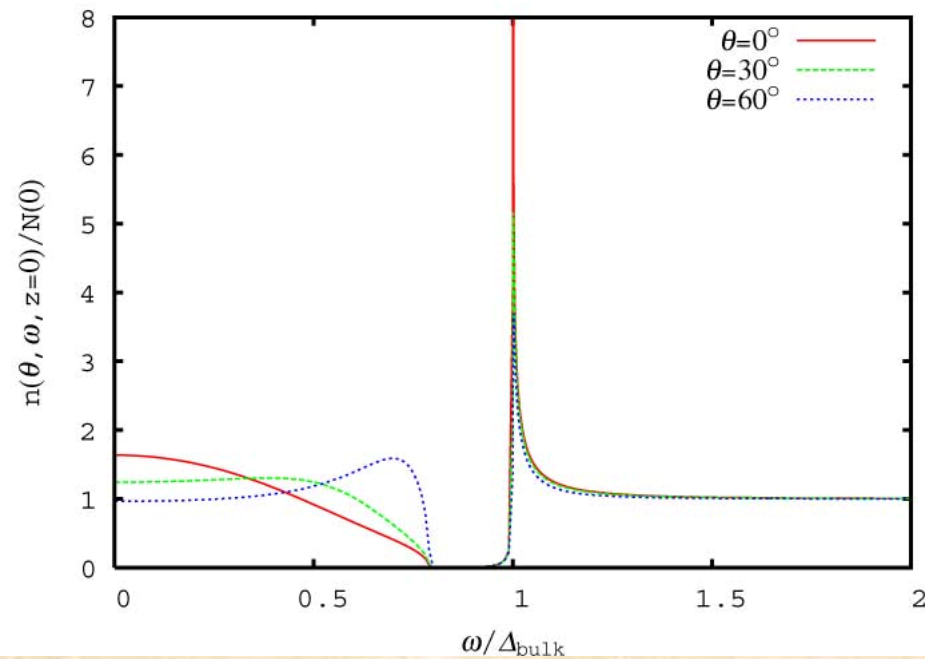


$W=0$

Density of States at Surface with Roughness

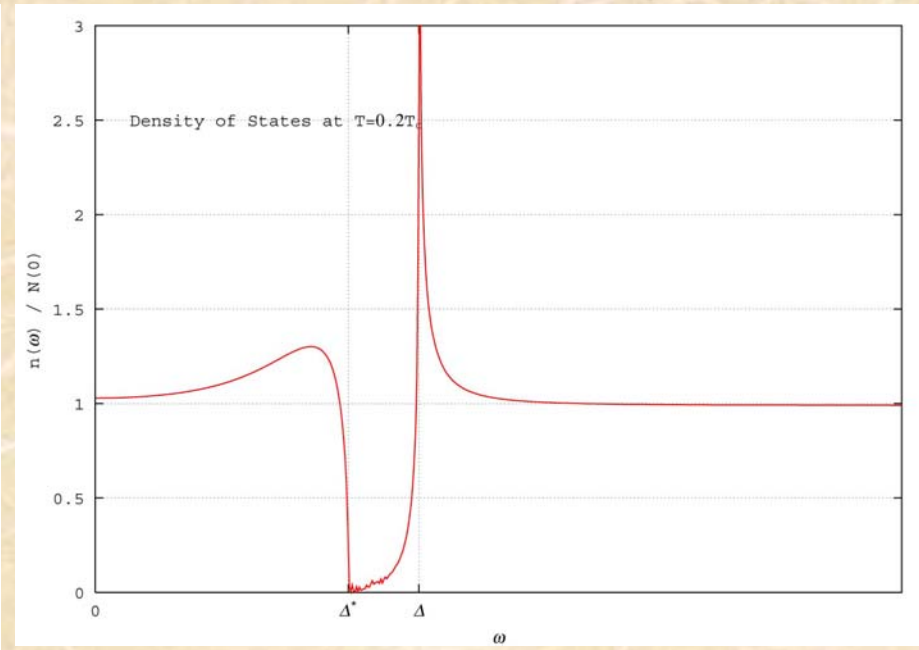
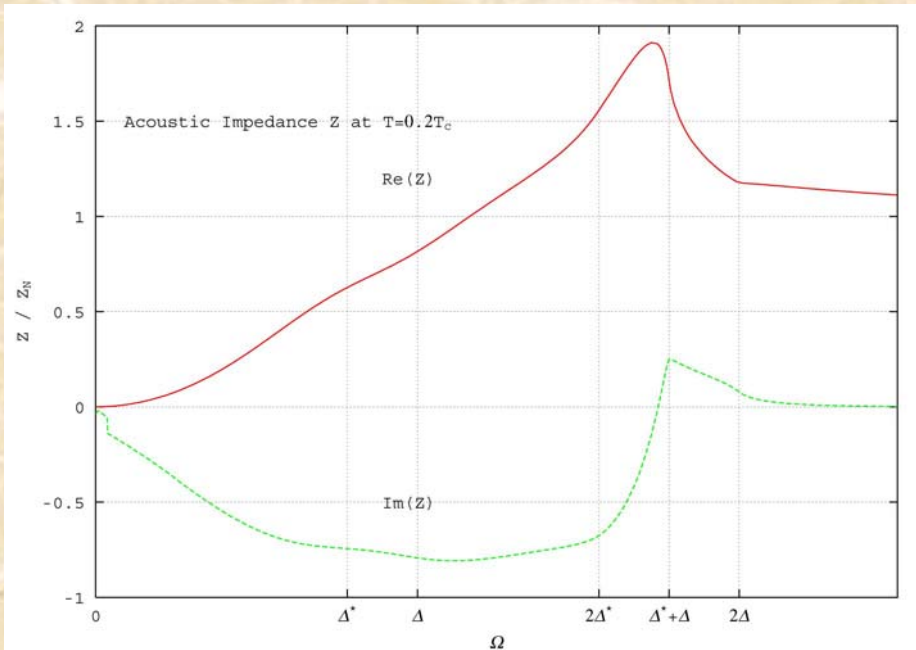


$W=0.1$



$W=1.0$

Frequency Dependence of $Z=Z'+iZ''=P_{xz}/u_{\text{wall}}$

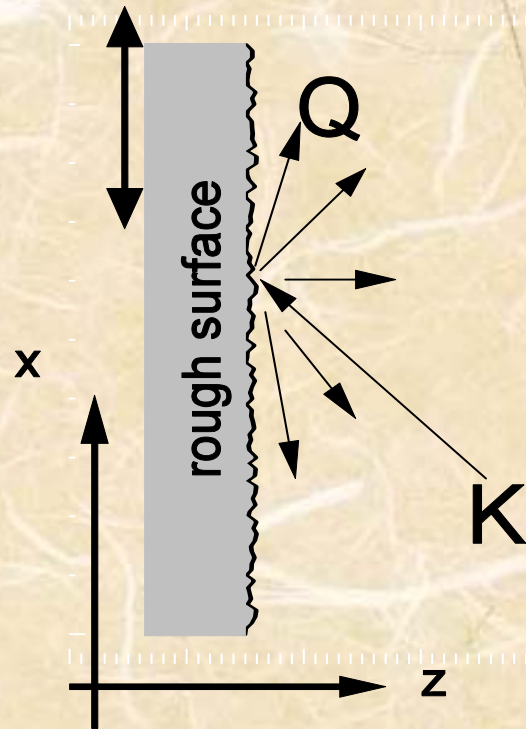


Structure in the acoustic impedance can be related to the pair excitations of midgap quasi-particle and propagating quasi-particle.

Acoustic Surface Spectroscopy

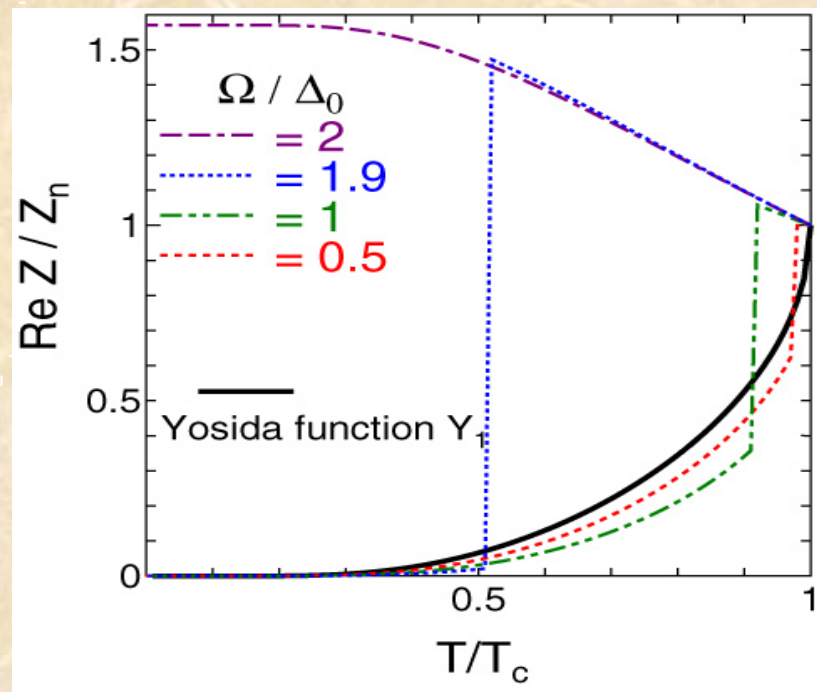
Boundary Condition for Oscillating Wall

$$u = dR(t) / dt$$



$$S_{KQ} \rightarrow e^{-iKR(t)} S_{KQ} e^{iQR(t')}$$

Analytic Result for s-wave pairing System



Unsolved Problems

1. Origin of the upper edge of the surface states band.
2. Fermi liquid effects

$$\frac{\partial}{\partial t} J_x(z) = -\frac{\partial}{\partial z} \Pi_{xz}$$

$$\Pi_{xz}(0) = -i\omega \int_0^\infty dz J_x(z)$$

$$J_x(z) = J_0(z) + F_1^s \int_0^\infty dz' K(z, z') J_x(z')$$

How the momentum of wall is transferred to the superfluid and how the transverse wave motion develops as it is away from the wall?

Sound Propagation in Liquid ^3He in Aerogel

Aerogel SiO_2 の鎖状ネットワーク

エアロジェル (98%) の特徴的な長さ

- SiO_2 粒子 $a=3\text{nm}$, $k_F a \sim 20$
- 平均鎖間距離 35nm
- フラクタル相関長 100nm
フラクタル次元 $D = 1.8$, $3 - D = 1.2 > 1$
- ^3He の平均自由行程 200nm
- ^3He のコヒーレンス長 $15\text{nm} \sim 80\text{nm}$

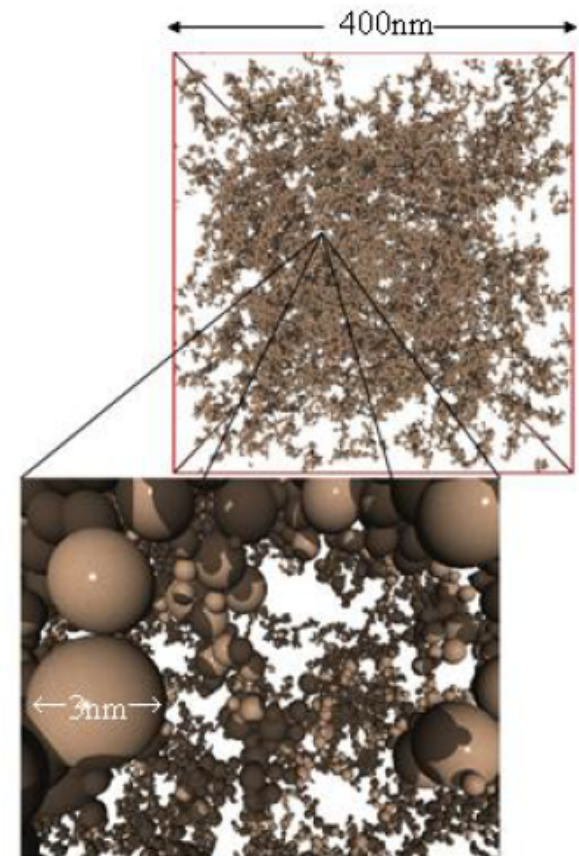
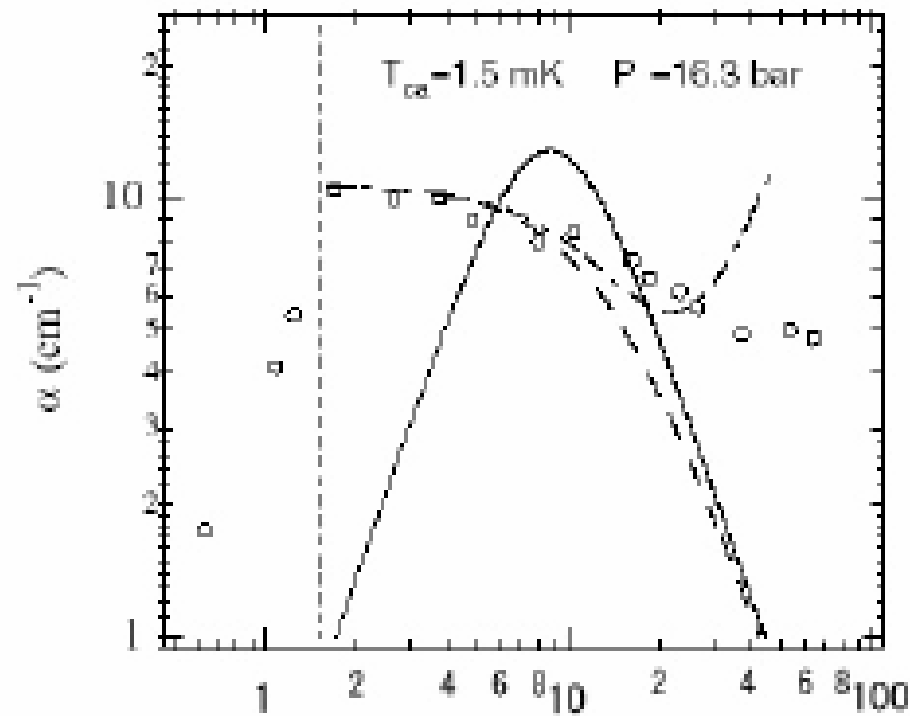


図: シミュレーションにより描かれた
エアロジェルの構造
(<http://spindry.phys.nwu.edu/DLCA>)

Sound Propagation

Nomura et al. Phys. Rev. Lett, 85, 4325 (2000).

15MHz, $\lambda \sim 20\mu\text{m}$



Momentum Conservation Law in Aerogel

$$\frac{\partial J_i}{\partial t} + \frac{\partial \Pi_{ij}}{\partial x_j} + F_i = 0$$

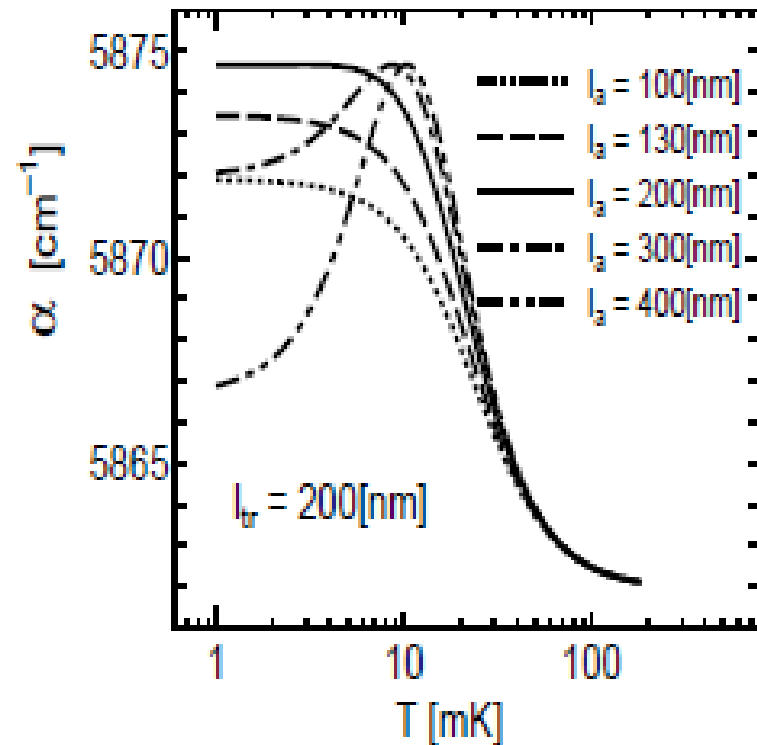
$$\Pi_{ij} \leftarrow \eta$$

Viscoelastic Model

$$\eta \rightarrow \eta(\omega) = \frac{\eta}{1 + \omega\tau}$$

$$\frac{1}{\tau} = \frac{1}{\tau_\eta} + \frac{1}{\tau_a}$$

$$F_i = \frac{1 + F_1^S/3}{\tau_{tr}} J_i$$



Aerogel Motion

Equation of motion of aerogel

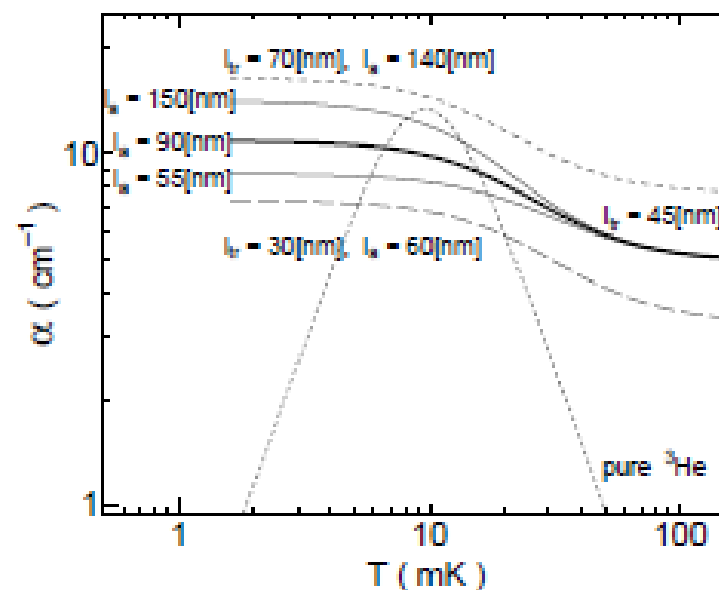
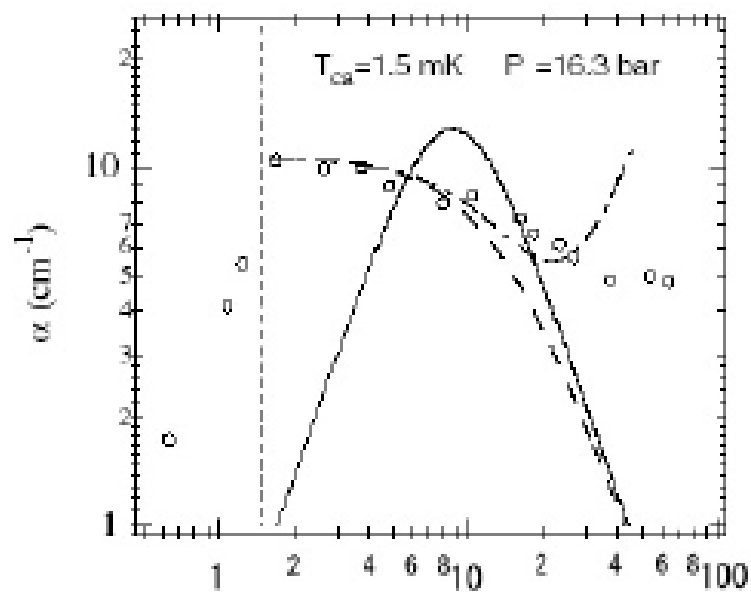
$$\rho_a \left(\frac{\partial^2}{\partial t^2} \mathbf{u}_a - c_a^2 \Delta \mathbf{u}_a \right) = \mathbf{F}, \quad \mathbf{v}_a = \frac{\partial}{\partial t} \mathbf{u}_a$$

Collision drag force (Pippard, Holstein)

$$\mathbf{F} = - \sum_{\mathbf{p}} \mathbf{p} I_{\text{imp}} = \frac{1 + F_1^S/3}{\tau_{\text{tr}}} \rho (\mathbf{v} - \mathbf{v}_a)$$

$$\begin{aligned} \mathbf{J} &= \rho \mathbf{v} \\ \frac{\partial J_i}{\partial t} &= - \frac{\partial \Pi_{ij}}{\partial x_j} - F_i \end{aligned}$$

Ichikawa et al. JPSJ 70,3483 (2001).



$$\omega = 2\pi \times 15 \text{ MHz}, \quad \omega T_{\text{tr}} \sim 0.7$$

Electronic Raman Scattering from Sr_2RuO_4

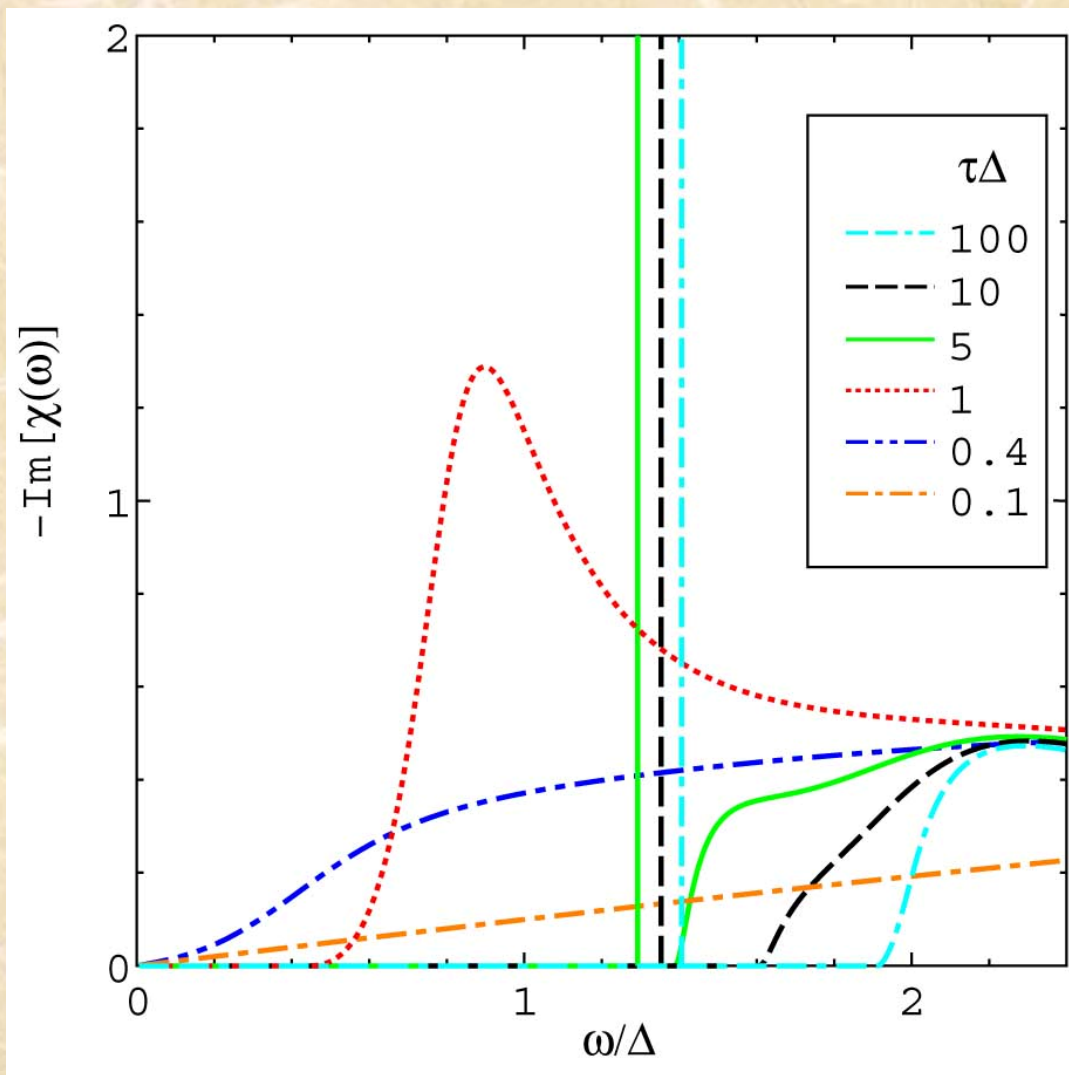
- Pair excitations
- Order Parameter Collective Modes

$$k_x \pm ik_y$$

$$\sin k_x \pm i \sin k_y$$

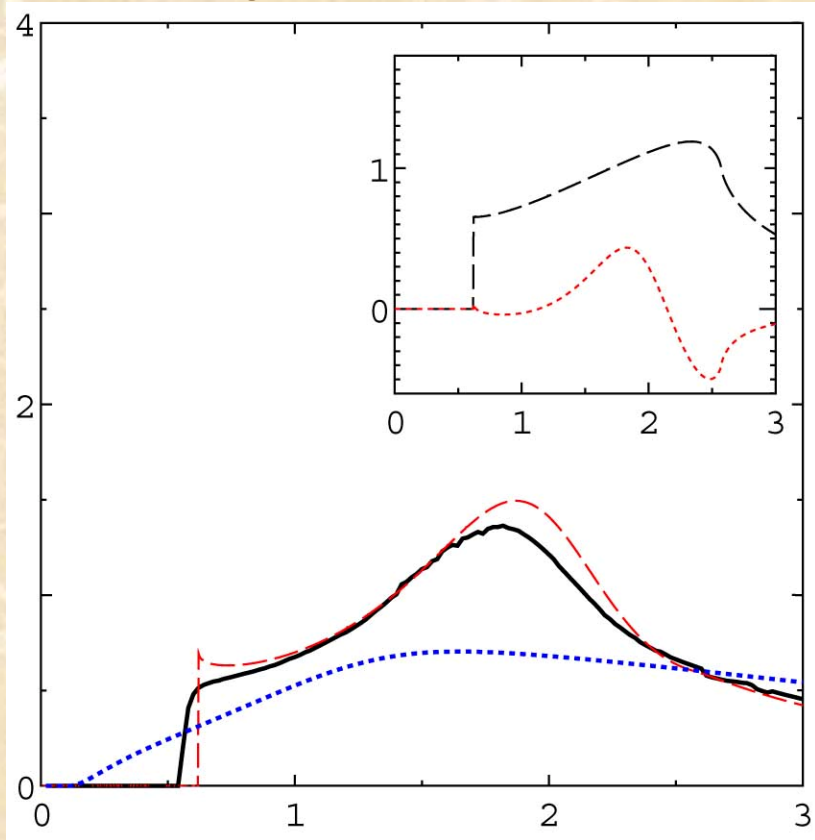
c.f. clapping mode in $^3\text{He-A}$

$$k_x + ik_y$$

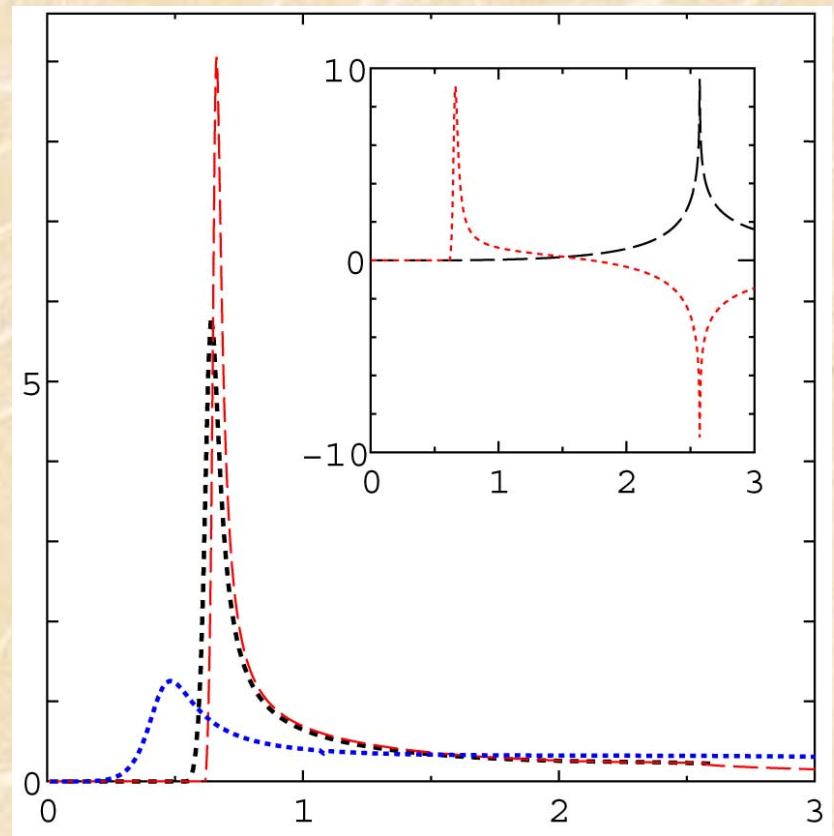


$$\sin k_x + i \sin k_y$$

B_{1g} mode



B_{2g} mode



$$\tau\Delta = \infty, \tau\Delta = 10, \tau\Delta = 1$$

Future Problems to Study

- quantitative study of acoustic impedance of superfluid ^3He
- dynamics of systems with boundary optical conductivity of superconductors contacts
- proximity effects of superfluid ^3He in aerogel in contact with pure superfluid ^3He