# Effect of Spin-Orbit Interaction in Spin-Triplet Superconductor: Structure of d-vector and Anomalous $O^{17}$-NQR Relaxation in $Sr_2RuO_4$


K. Miyake and H. Kohno

Osaka University

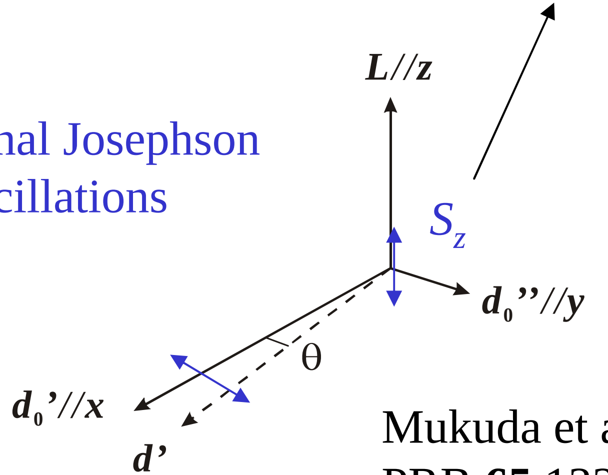

Anomalous relaxation of NQR in *z*-direction

Free energy $F_{so}$ for SO coupling:

$$F_{so} = -g_{so}(\mathrm{i}\boldsymbol{d}\times\boldsymbol{d}^{*})\cdot\boldsymbol{L}$$

$$g_{so} = \mu_B^2(m/m^*)6\pi\Psi^2 V$$

Internal Josephson oscillations



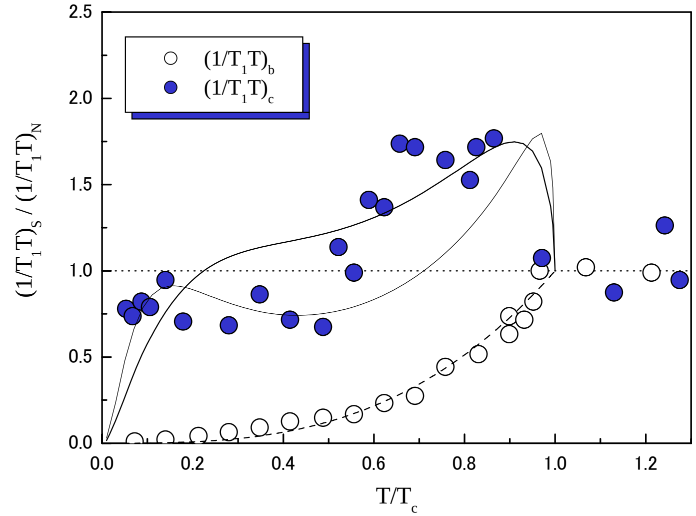

Leggett-Takagi (1977)

Estimation of $g_{so}$ in GL region:

$$g_{so} = |F_{cond}|\,\mu_B^2(m/m^*)12\pi N_F \times [\ln(1.14\beta_c\varepsilon_c)]^2\,(1-T/T_c)^{-1}$$

$F_{cond}$: condensation energy

Mukuda et al 2002 PRB **65** 132507




$d$-vector $\boldsymbol{d}_0$ in equilibrium state:

$$d_{0x} = (1+\eta^2)^{-1/2},\ d_{0y} = \mathrm{i}\eta(1+\eta^2)^{-1/2},\ d_{0z} = 0$$

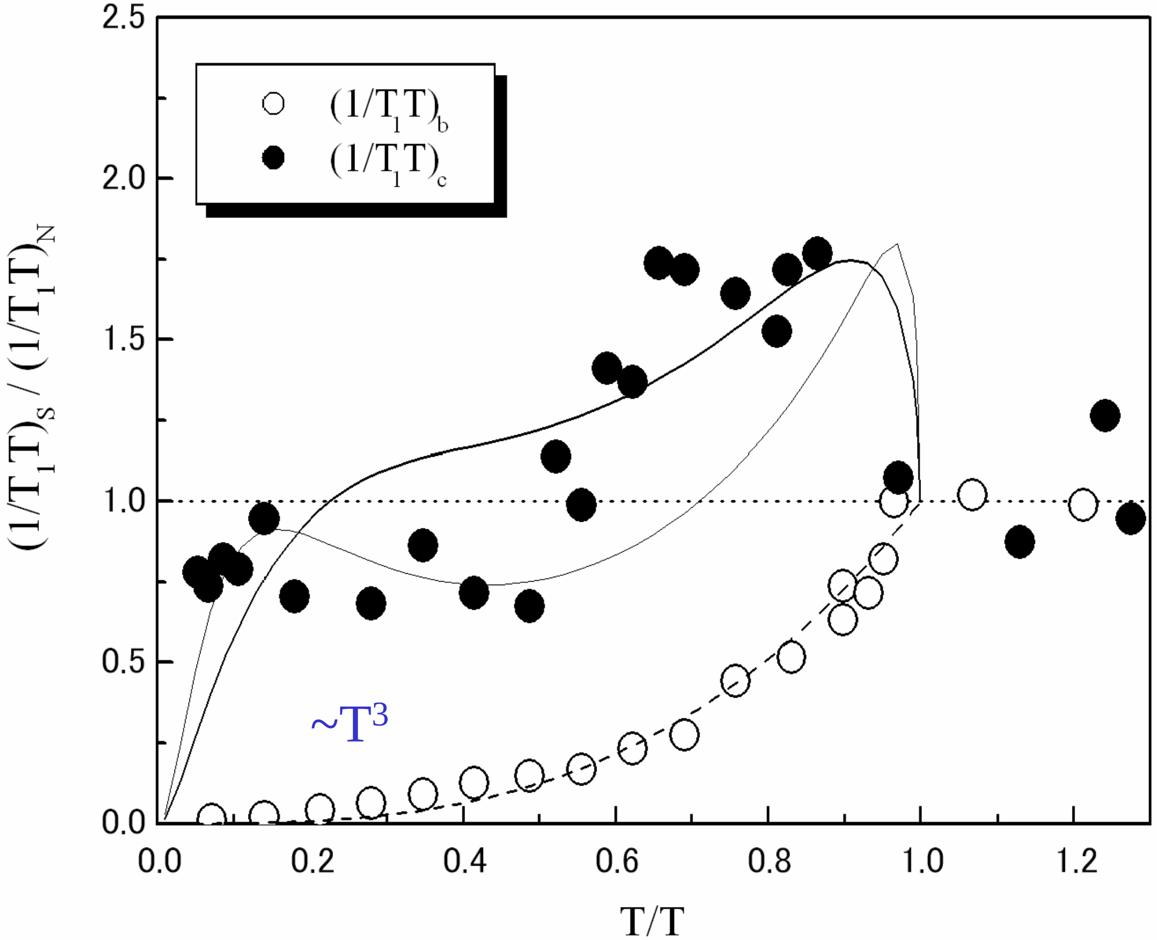
$\eta = g_{so}/4|F_{cond}|$ : weakly non-unitary

# Anomalous NQR relaxation of $^{17}O$

H. Mukuda et al : Phys. Rev B **65** (2002) 132507

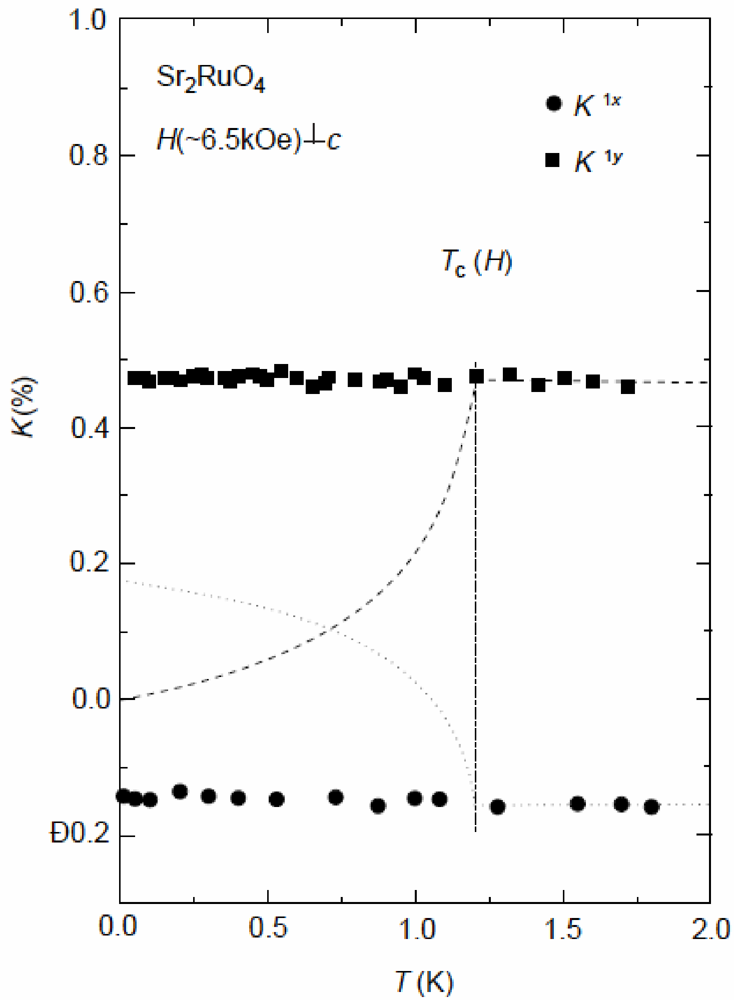
$$(1/T_1T)_i \propto \sum_q \frac{\mathrm{Im}\chi_i(\mathbf{q},\omega)}{\omega}, \qquad (i = a, b, c)$$

H = 0



Knight shift: unchanged across Tc → Evidence for ESP pairing

K. Ishida et al : Nature **396** (1998) 658




$$\mathbf{d} \parallel \hat{c}, \qquad \mathbf{H} \perp \hat{c}$$

Oscillations of **d**-vector gives spin fluctuations in the plane

Inconsistent with NQR relaxation

**d**-vector rotated by magnetic field ?

or

intrinsic ?

# Knight shift: unchanged across Tc also for **H//c**

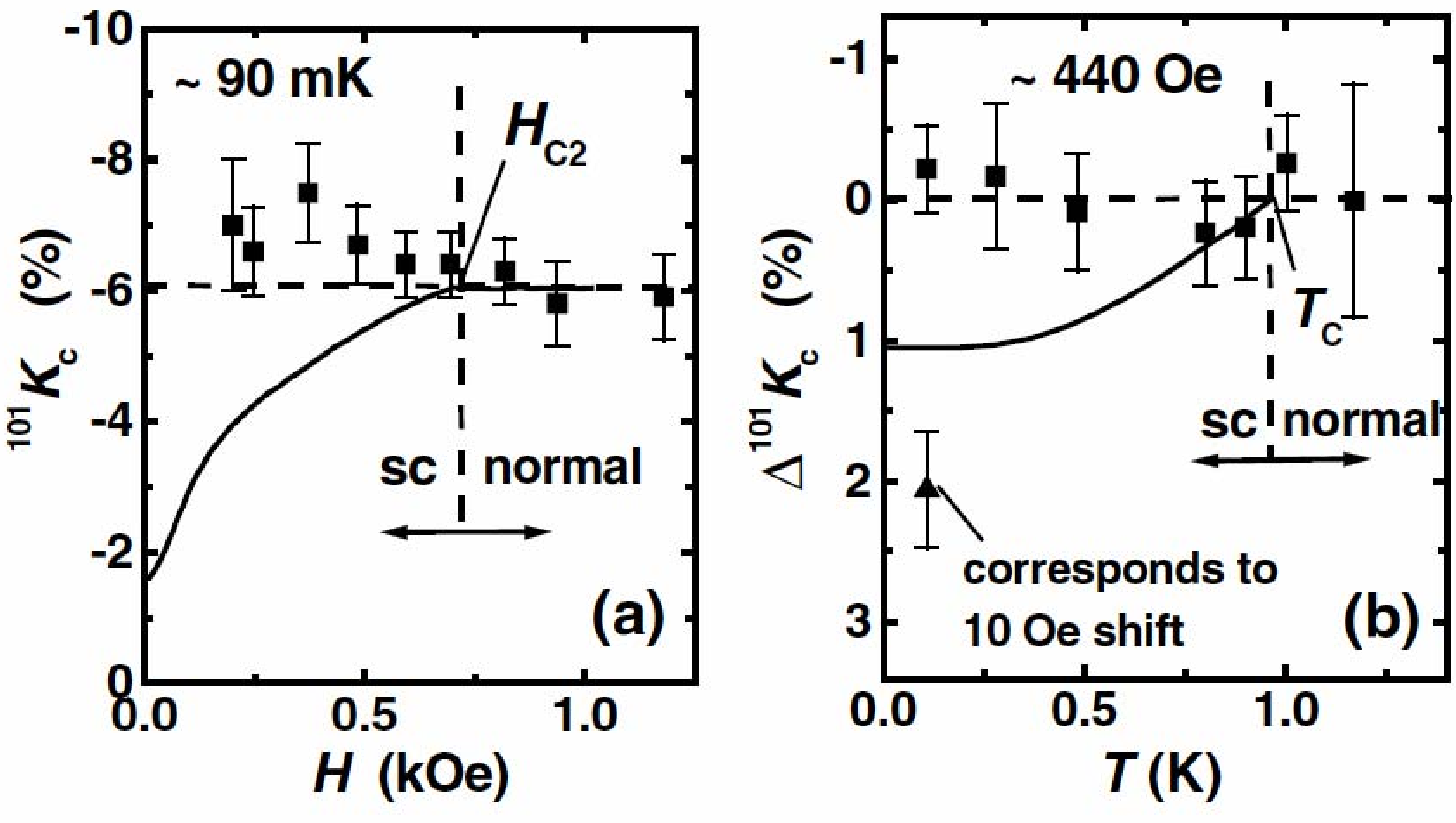
H. Murakawa et al : Phys. Rev. Lett. **93**, 167004 (2004)

$$\mathbf{d} \perp \hat{c},\ \mathbf{H} \parallel \hat{c}$$

**d**-vector rotated by magnetic field ?

or

intrinsic ?



# Spin-Orbit Interaction Hamiltonian

$$H_{\mathrm{so}} = -\frac{\mu_{\mathrm{B}}^2}{\hbar}\frac{m}{m^*}\sum_i \sum_{j\neq i} \frac{1}{r_{ij}^3}\vec{\sigma}_i \cdot \left[\vec{r}_{ij} \times [(2\bar{g}-1)\vec{p}_i - 2\bar{g}\vec{p}_j]\right], \qquad \bar{g} \equiv \mu_{\mathrm{eff}}/\mu_{\mathrm{B}}$$

$$\begin{aligned} H_{\mathrm{so}} &= -\frac{\mu_{\mathrm{B}}^2}{\hbar}\frac{m}{m^*}\int\int \mathrm{d}\mathbf{R}\mathrm{d}\mathbf{r}\frac{1}{r^3}\psi_\alpha^\dagger(\mathbf{R}+\mathbf{r}/2)\psi_\gamma^\dagger(\mathbf{R}-\mathbf{r}/2)\vec{\sigma}_{\alpha\beta}\delta_{\gamma\delta} \\ &\qquad \cdot[\vec{r}\times(-\mathrm{i}\hbar)((4\bar{g}-1)\vec{\nabla}_r - \frac{1}{2}\vec{\nabla}_R)]\psi_\delta(\mathbf{R}-\mathbf{r}/2)\psi_\beta(\mathbf{R}+\mathbf{r}/2). \end{aligned}$$

## Decoupling approximation

$$\begin{aligned} &\langle\psi_\alpha^\dagger(\mathbf{R}+\mathbf{r}/2)\psi_\gamma^\dagger(\mathbf{R}-\mathbf{r}/2)\psi_\delta(\mathbf{R}-\mathbf{r}/2)\psi_\beta(\mathbf{R}+\mathbf{r}/2)\rangle \\ &\qquad \simeq \langle\psi_\alpha^\dagger(\mathbf{R}+\mathbf{r}/2)\psi_\gamma^\dagger(\mathbf{R}-\mathbf{r}/2)\rangle\langle\psi_\delta(\mathbf{R}-\mathbf{r}/2)\psi_\beta(\mathbf{R}+\mathbf{r}/2)\rangle \\ &\qquad = \langle\psi_\alpha^\dagger(\mathbf{r}/2)\psi_\gamma^\dagger(-\mathbf{r}/2)\rangle\langle\psi_\delta(-\mathbf{r}/2)\psi_\beta(\mathbf{r}/2)\rangle. \end{aligned}$$

## Free energy

$$F_{\mathrm{so}} \equiv \langle H_{\mathrm{so}}\rangle = -\frac{\mu_{\mathrm{B}}^2}{\hbar}\frac{m}{m^*}(4\bar{g}-1)V\int \mathrm{d}\mathbf{r}\frac{1}{r^3}\vec{\sigma}_{\alpha\beta}\delta_{\gamma\delta}\cdot F_{\gamma\alpha}^*(\mathbf{r})[\vec{r}\times(-\mathrm{i}\hbar)\vec{\nabla}_r)]F_{\delta\beta}(\mathbf{r}),$$

$$F_{\alpha\beta}(\mathbf{r}) \equiv \langle\psi_\alpha(\mathbf{r}/2)\psi_\beta(-\mathbf{r}/2)\rangle. = \mathrm{i}(\vec{\sigma}\sigma_2)_{\alpha\beta}\cdot\vec{F}(\mathbf{r}),$$

## 2d ABM state

$$F_{\mathrm{so}} = -g_{\mathrm{so}}(\mathrm{i}\vec{d}\times\vec{d}^*)\cdot\vec{L}, \qquad g_{\mathrm{so}} = \mu_{\mathrm{B}}^2\frac{m}{m^*}(4\bar{g}-1)4\pi\Psi^2 V$$

# GL region : Non-Unitary State due to Spin-Orbit Coupling

$$F_{\rm GL} = \frac{1}{2}\left(\frac{{\rm d}n}{{\rm d}\epsilon}\right)\left[-\left(1-\frac{T}{T_{\rm C}}\right)\frac{\Delta_\uparrow^2+\Delta_\downarrow^2}{2}+\frac{7\zeta(3)}{16}\frac{\kappa}{(\pi k_{\rm B}T_{\rm C})^2}\frac{\Delta_\uparrow^4+\Delta_\downarrow^4}{2}\right],$$

$\Delta_\uparrow = \Delta_\downarrow = \Delta$ (unitary state)

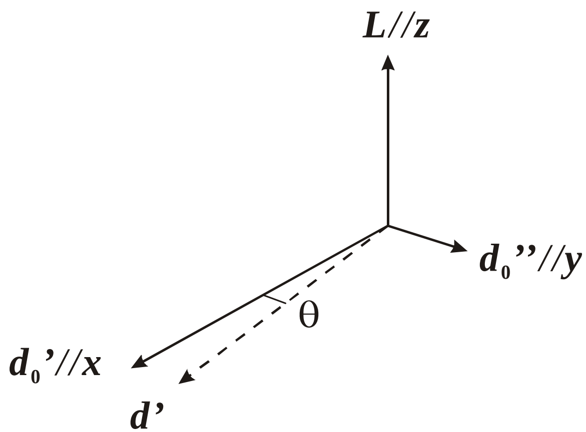
$$F_{\rm cond}^{\rm unit} = -\frac{1}{4}\left(\frac{{\rm d}n}{{\rm d}\epsilon}\right)\frac{8}{7\zeta(3)}\frac{1}{\kappa}(\pi k_{\rm B}T_{\rm C})^2\left(1-\frac{T}{T_{\rm C}}\right)^2,$$

$$\frac{g_{\rm SO}}{|F_{\rm cond}^{\rm unit}|} = \frac{m}{m^*}\times(4\bar{g}-1)4\pi\mu_{\rm B}^2\left(\frac{{\rm d}n}{{\rm d}\epsilon}\right)[\ln(1.14\beta_{\rm C}\epsilon_{\rm C})]^2\left(1-\frac{T}{T_{\rm C}}\right)^{-1}$$

$$\hat{\Delta} = \frac{\Delta_0}{\sqrt{1+\eta^2}}\begin{pmatrix} -1-\eta & 0 \\ 0 & 1-\eta \end{pmatrix}$$

$$d_{0x} = \frac{1}{\sqrt{1+\eta^2}}, \qquad d_{0y} = {\rm i}\frac{\eta}{\sqrt{1+\eta^2}}, \qquad d_{0z} = 0$$

$$\eta = \frac{g_{\rm SO}}{4|F_{\rm cond}^{\rm unit}|} \simeq \frac{1}{4}\times 1.5\times 10^{-3}\left(1-\frac{T}{T_{\rm C}}\right)^{-1}$$

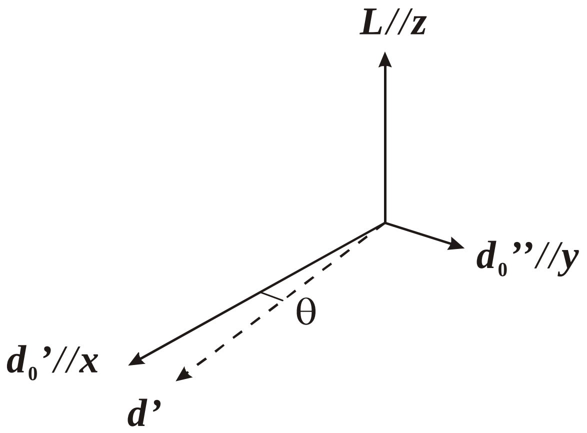



$\bar{g} \simeq 1$, $({\rm d}n/{\rm d}\epsilon) = m^*/c\pi\hbar^2$, with $c = 6.4\AA$,
$m^*/m_{\rm band} \simeq 12$, $1.14\beta_{\rm C}\epsilon_{\rm C} \simeq 20$

# Internal Josephson Effect

cf. $^3$He-A, Leggett (1974)

$$\hat{\Delta} \simeq \Delta_0 \begin{pmatrix} (-1-\eta)e^{-\mathrm{i}\theta} & 0 \\ 0 & (1+\eta)e^{\mathrm{i}\theta} \end{pmatrix}$$




$$(\mathrm{i}\vec{d} \times \vec{d}^*)_z = 2(\vec{d}' \times \vec{d}'')_z = 2\eta \cos\theta$$

$$F_{\mathrm{SO}}(\theta) = -g_{\mathrm{SO}}(2\eta)\cos\theta$$

## Coupled equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}(N_\uparrow - N_\downarrow) = -\frac{\partial F_{\mathrm{SO}}}{\partial(2\theta)} = -g_{\mathrm{SO}}\eta \sin\theta$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(2\theta) = 2\Delta\mu = \frac{2(\mu_{\mathrm{B}}/\hbar)^2}{\chi_z}(N_\uparrow - N_\downarrow)$$

## Eigen frequency

$$\Omega^2 = \frac{g_{\mathrm{SO}}(\mu_{\mathrm{B}}/\hbar)^2}{\chi_z}\eta = \frac{g_{\mathrm{SO}}(\mu_{\mathrm{B}}/\hbar)^2}{\chi_z}\frac{g_{\mathrm{SO}}}{4|F_{\mathrm{cond}}|}$$

$$\Omega \simeq 6.9 \times 10^7 \times \frac{T_{\mathrm{c}}}{\sqrt{\kappa}} \ln(1.14\beta_{\mathrm{c}}\epsilon_{\mathrm{c}}) \quad \mathrm{sec}^{-1}$$

Comparable to NQR frequency

NQR Relaxation Rate : by applying Leggett &Takagi theory

Ann. Phys. **106** (1977) 79

$$\chi_z(\omega) = -\frac{\Omega^2 \chi_z}{\omega^2 - \Omega^2 + \mathrm{i}\Gamma\omega}$$

$$\Gamma = \gamma_0 \tau \Omega^2, \quad \tau = b\frac{\hbar T_\mathrm{F}}{k_\mathrm{B} T^2} = 7.6 \times 10^{-12} \times b\frac{T_\mathrm{F}}{T^2}$$

Life time of quasiparticles in normal state

$$\gamma_0 \equiv [1 - Y(T)]^{-1} Y(T) \frac{\chi_z}{\mu_\mathrm{B}^2 (\frac{\mathrm{d}n}{\mathrm{d}\epsilon})}$$

$Y(T)$ : Yosida function

$$\frac{1}{T_1 T} = A \sum_{q<q_\mathrm{c}} \frac{\mathrm{Im}\chi_z(q,\omega)}{\omega}, \qquad \frac{\mathrm{Im}\chi_z(\omega)}{\omega} = \chi_z \frac{\gamma_0\tau}{[(\frac{\omega}{\Omega})^2 - 1]^2 + (\gamma_0\omega\tau)^2}$$

$$q_\mathrm{c} \sim r(\pi/\xi_0), \; r \; (<1)$$

$$\frac{1}{T_1 T} = A\chi_z \frac{n_{2d}}{c} r^2 (\frac{a}{\xi_0})^2 \frac{\gamma_0\tau}{1 + (\gamma_0\omega\tau)^2}$$

$$\frac{\xi_0}{a} = 1.1 \times 10^{-1} \frac{T_\mathrm{F}}{T_\mathrm{c}}$$

cylindrical or 2d spherical model

$$(\frac{\mathrm{d}n}{\mathrm{d}\epsilon}) \simeq \frac{1}{c}\frac{n_{2d}}{k_{\mathrm{B}}T_{\mathrm{F}}}$$

NQR relaxation rate

$$\frac{1}{T_1 T} = A \times 6.5 \times 10 \times \frac{cn_{\mathrm{L}}}{n_{2d}} br^2 \chi_z \hbar (\frac{\mathrm{d}n}{\mathrm{d}\epsilon})(\frac{T_{\mathrm{c}}}{T})^2 \frac{\gamma_0}{1 + (\gamma_0 \omega\tau)^2}$$

Korringa law in normal state

$$(\frac{1}{T_1 T})_{\mathrm{K}} = A \frac{1}{4} \chi_z \frac{\hbar (\frac{\mathrm{d}n}{\mathrm{d}\epsilon})}{1 + F_0^{\mathrm{a}}}$$

$$\gamma_0 \simeq 2.0 \times [1 - Y(T)]^{-1} Y(T)$$

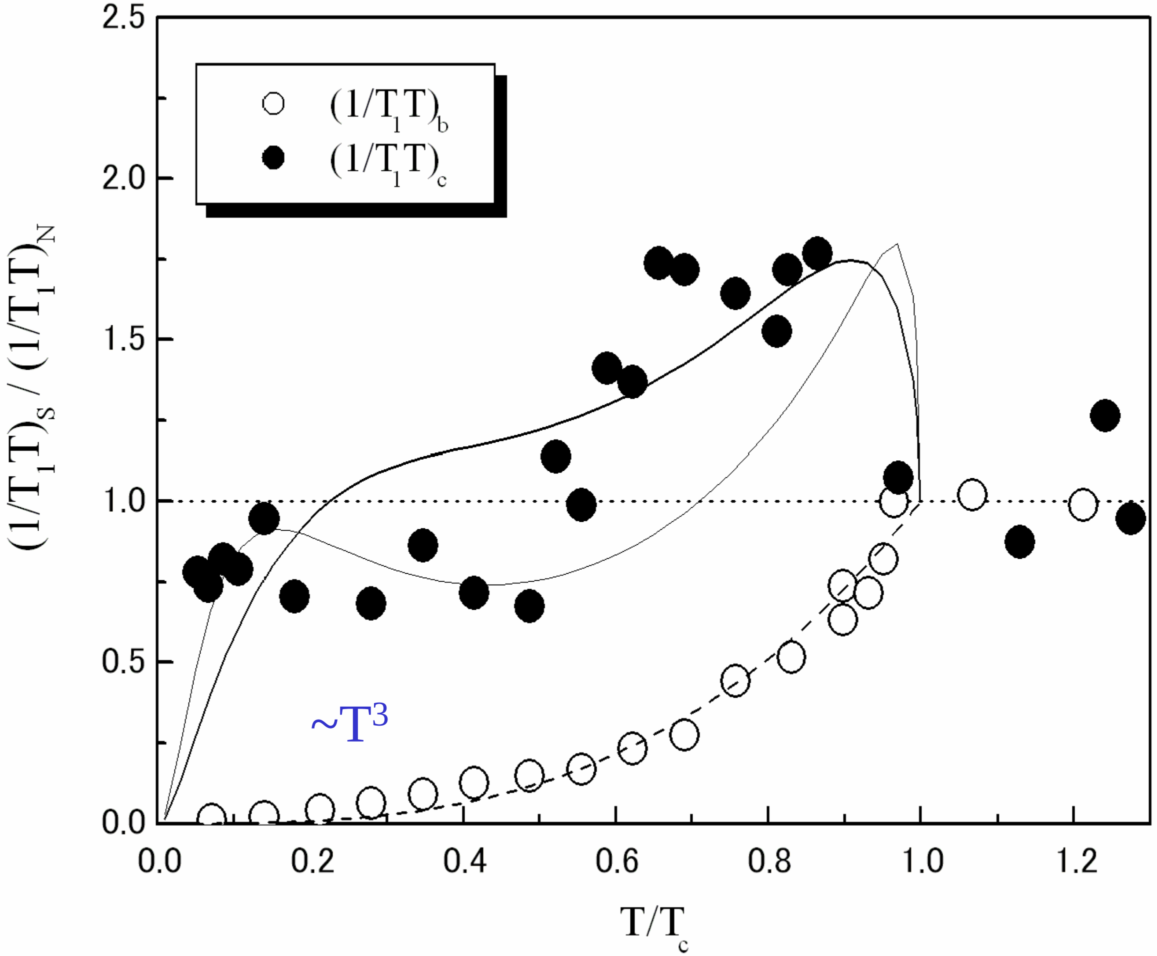
$$\omega\tau \simeq 7.6 \times 10^{-5} \times b \frac{T_{\mathrm{F}}}{T^2}$$

# Anomalous NQR relaxation of $^{17}$O

H. Mukuda et al : Phys. Rev B **65** (2002) 132507

$$(1/T_1T)_i \propto \sum_q \frac{\mathrm{Im}\chi_i(\mathbf{q},\omega)}{\omega}, \quad (i = a, b, c)$$

H = 0



# Summary

Anomalous NQR relaxation of $Sr_2RuO_4$ can be explained qualitatively by internal Josephson effect arising from spin-orbit coupling effect on spin-triplet Cooper pairs with d-vector in the plane.

It is rather hard to explain the anomalous relaxation if the d-vector is along the c-direction.

# Prospect

- Search for hydrodynamic collective mode
  cf. collisionless collective modes in superfluid $^3He$
- New type of anisotropic superconductivity
- New type of superfluidity, quartet, sextet, etc