Quantum Turbulence in Superfluid ^{4}He Generated by a Vibrating Wire

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OUTLINE

Flow of superfluid ^{4}He generated by a vibrating wire

Motivations

- How dose the flow around the wire develop into turbulence ?
- Vortex dynamics in the ac flow of superfluid ^{4}He

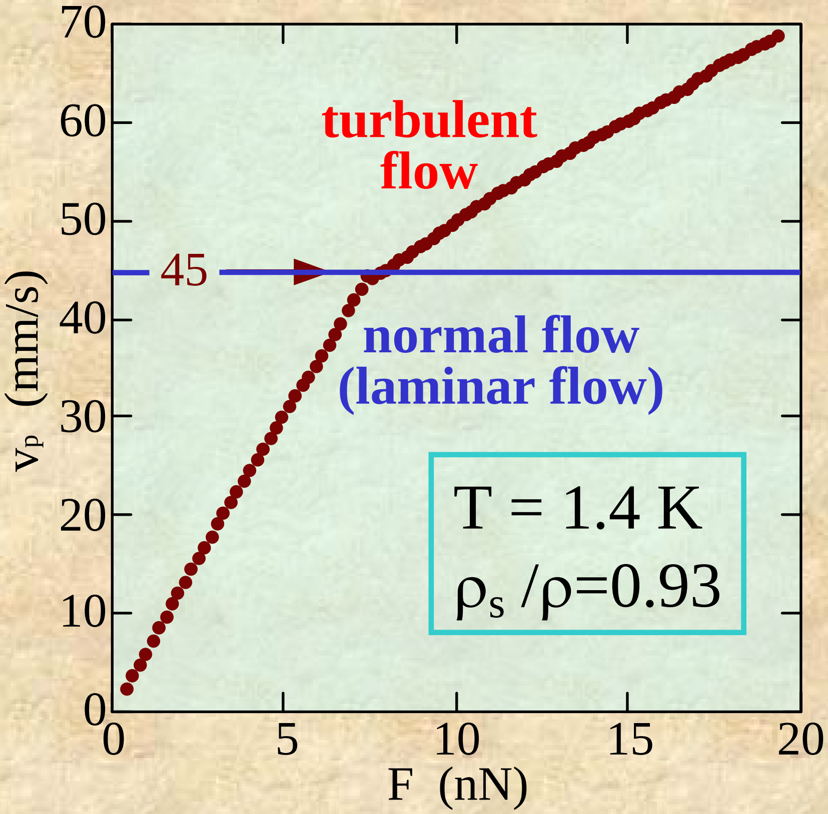
Response of the vibrating wire at low temperatures

- Resonance frequency ⇨ Effective wire diameter
- Vortex dynamics
- Switching behavior

Critical velocity

- Temperature dependence ⇨ Phase diagram

Response of a Vibrating Wire in Superfluid ^{4}He

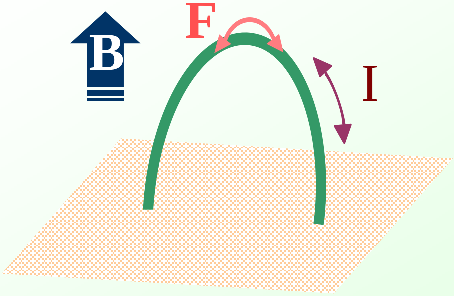


The laminar flow and the turbulent flow are clearly separated at a critical velocity v_c.

v_c = 45 mm/s << 60 m/s

(: vortex nucleation velocity)

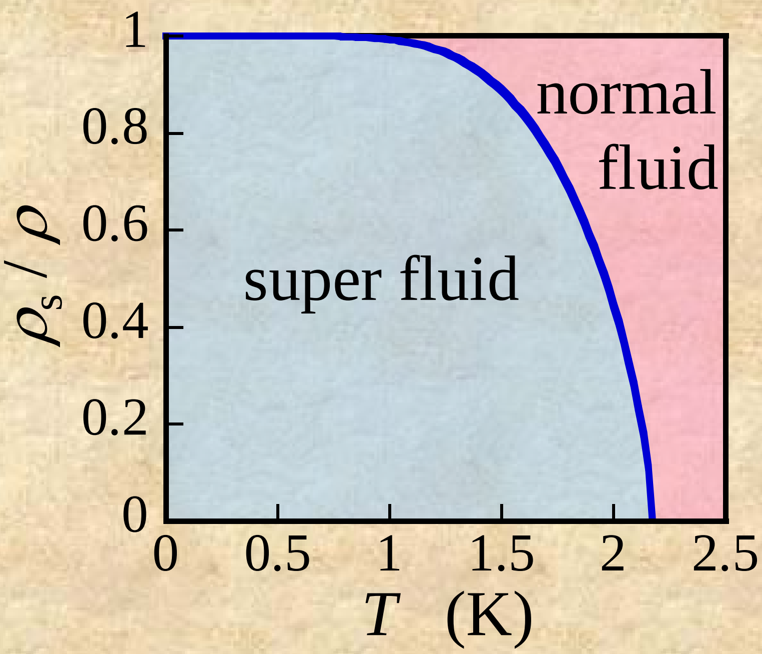
Vibrating Wire



NbTi wire
(ϕ 13μm)

Superfluid Helium-4

- order parameter: $\Psi = \phi e^{i\varphi}$
- density: $\rho_s / \rho = |\Psi|^2$
- velocity: $v_s = (\hbar / m) \nabla \varphi$

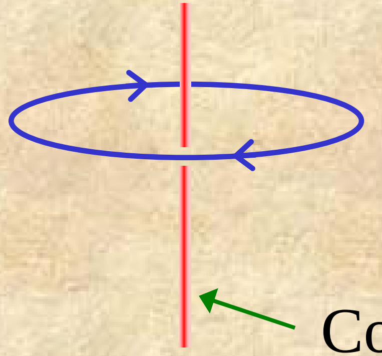


Quantized Vortex

$$(\hbar / m) \oint_C \nabla \varphi \cdot dl$$

$$= (\hbar / m) 2\pi n \quad (n = 0, 1, 2, \cdots)$$

$$\kappa = (\hbar / m) 2\pi = 9.97 \times 10^{-8} \, \mathrm{m}^2/\mathrm{s}$$



$$v_s = \frac{\kappa}{2\pi r}$$

Core: $\Psi = 0$
(radius ~ 0.1 nm)

- The core can be imagined as a thin string.

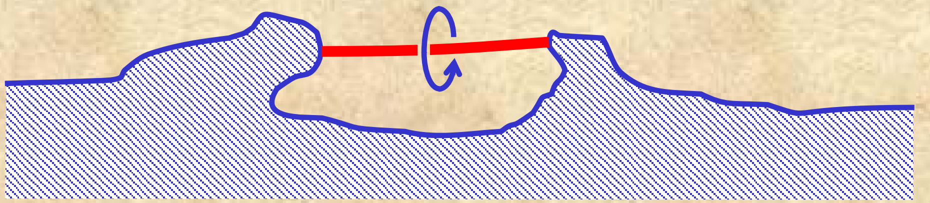
Vortex String in Superfluid Helium-4

- Quantized circulation prevents the core from becoming a superfluid.
- a **ring-shaped string** or a **string attached to a boundary**

- A ring string immediately disappears.

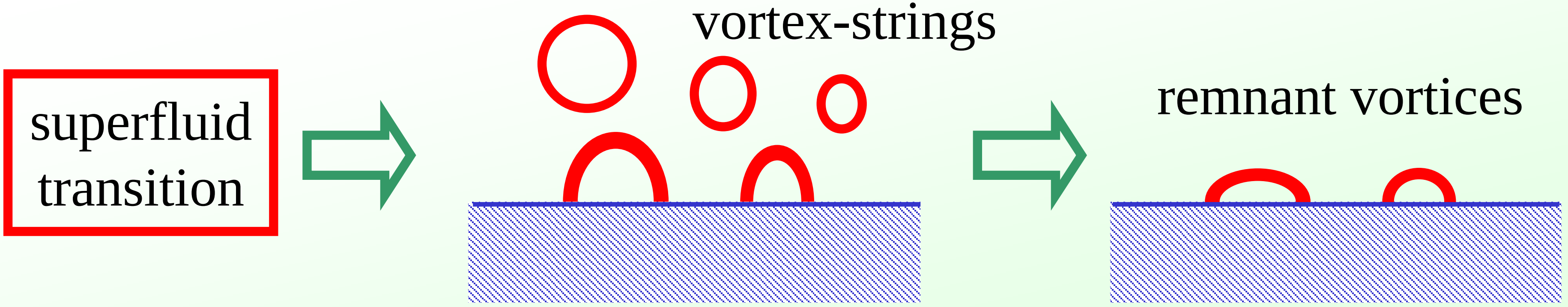
  (Interacting to normal fluid, Cascade process)

- A string still remains on a boundary.

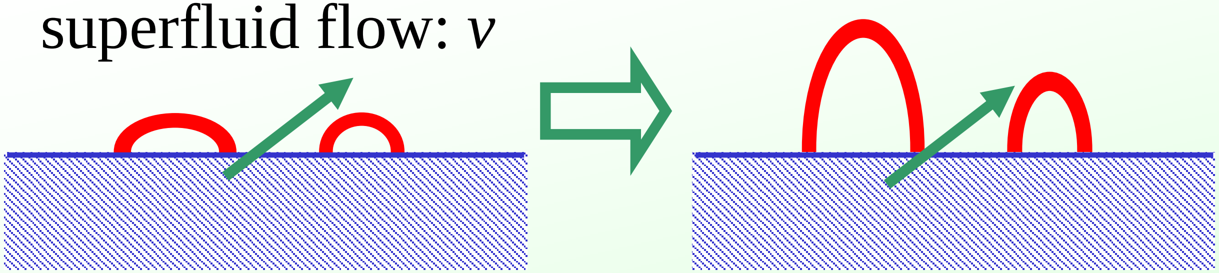


Possible explanation of the vortex creation in turbulence

1. Nucleation of vortex strings through the superfluid transition



2. Unstable expansion of a vortex string (Glaberson Donnelly instability)



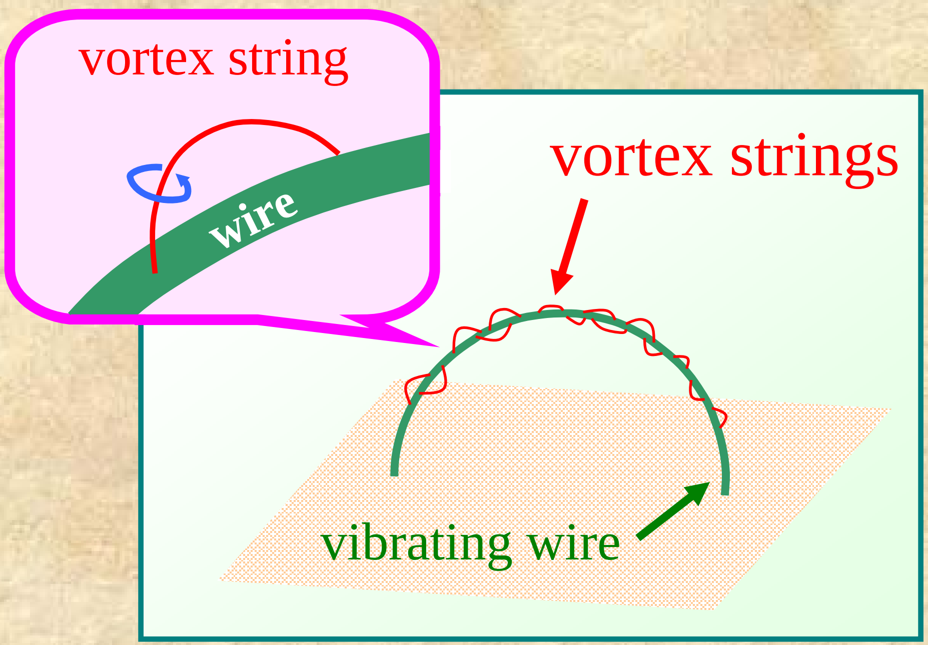
Vortex-strings unstably expand when $v > v_S$.

$$v_s \approx \frac{\kappa}{4\pi r_0}\left[\ln(8r_0/\xi) - \tfrac{1}{4}\right]$$

v_S = 45 mm/s → r_0 = 2.1 μm

(r_0 : radius of the string)

Possible image of vortex strings on a vibrating wire

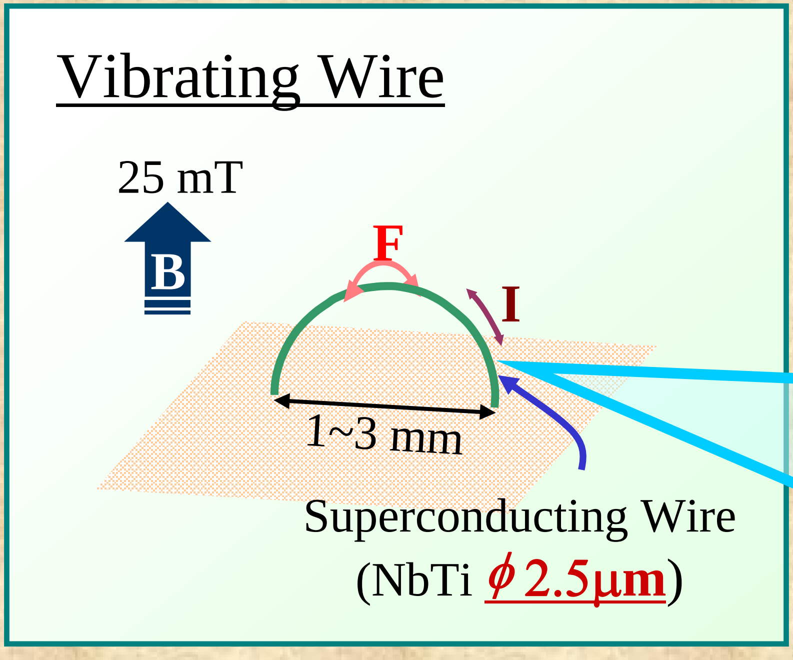


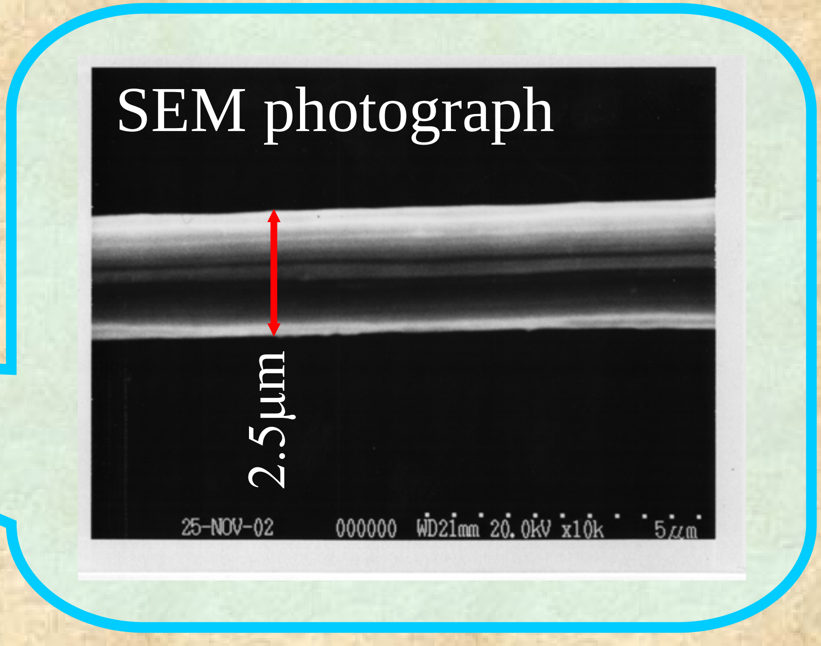
Motivation

- *How does the flow around the wire develop into turbulence?*
- *Vortex dynamics in the ac flow of superfluid ^{4}He.*

⇨ **The response of a vibrating wire at low temperatures (~ 30 mK).**

Vibrating wires for the turbulence study

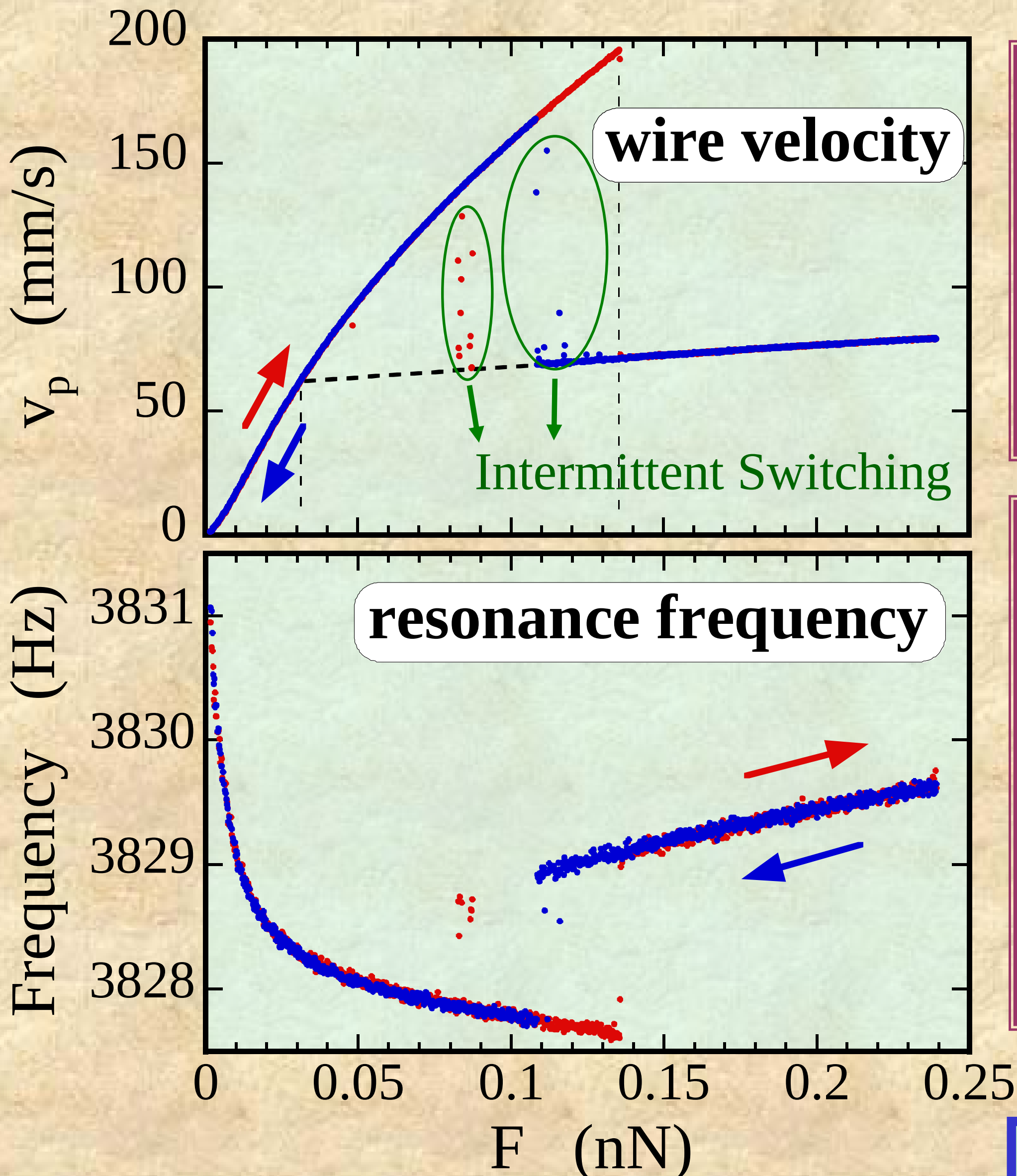




Vibrating Wires

prepared: 5 vibrating wires
shape: a half circle
wire diameters: ϕ 2.5 μm
resonance frequencies: 600 ~ 4,000 Hz
Q values: 1300 ~ 1700 (measured at 4.2 K in vacuum)

Velocity and Resonance Frequency at 30 mK



Velocity

- Response varies dramatically.
- The flow switches between Laminar flow ⇆ Turbulence

Resonance Frequency

- laminar flow ⇨ turbulence the frequency jumps upward.
- In the turbulent flow, the frequency increases.

Moment of inertia of the wire

Resonance Frequency with no normal fluid

$$\frac{f_{He}}{f_{vac}} = \left(\frac{\rho_w d_w^2}{(\rho_w + \rho_s) d_w^2} \right)^{1/2}$$

f_{He}, f_{vac} : frequency with and without He
ρ_w, ρ_s : wire and superfluid density
d_w : wire diameter (2.5 μm)

- at v_p = 60 mm/s

  f_{He} = 3828.3 Hz, f_{vac} = 3962.0 Hz, ρ_s = 0.14513 g/cm^3

  ⇨ ρ_w = 2.042 g/cm^3 < ρ_{NbTi} = 6.3 g/cm^3 **: too small !**

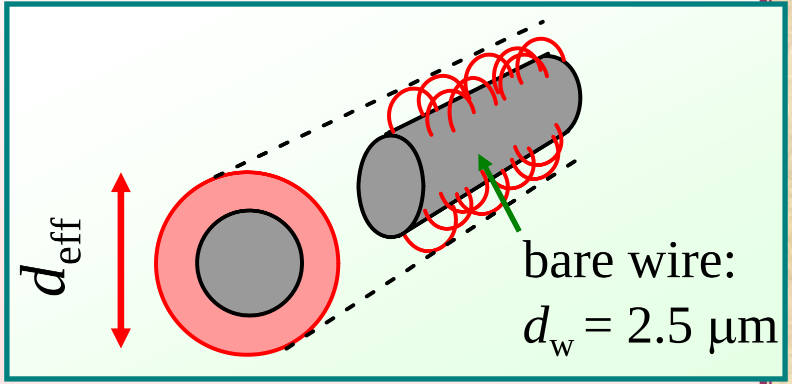
- Assuming: ***A vortex string behaves like a wall against a superfluid current.*** [Vinen *et al*, ***JLTP*** **135**, 423 (2004)]

  (d_{eff} : wire + vortex strings)

  ⇨ d_{eff} = 3.6 μm

  (d_w = 2.5 μm, ρ_w ~ 6.3 g/cm^3)

  (in He : $\rho_w d_w^2 + \rho_s (d_{eff}^2 - d_w^2) + \rho_s d_{eff}^2$)



FUTURE WORKS

Vortex dynamics in the ac flow of superfluid helium

- Vortex state on a boundary
  - Temperature, frequency, wire thickness, …
- Switching phenomenon between laminar and turbulent flow
  - Statistics analysis, …
- Dissipation process of vortices
- Vortex dynamics in superfluid helium-3
  - Vortex dynamics in ac flow generated by a vibrating wire
  - Vortex under rotation – Helsinki group