A03

Bose Superfluid and Quantized Vortex

New research of quantized vortices and superfluid turbulence

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Bose-Einstein condensation with internal degrees of freedom

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New research of quantized vortices and superfluid turbulence

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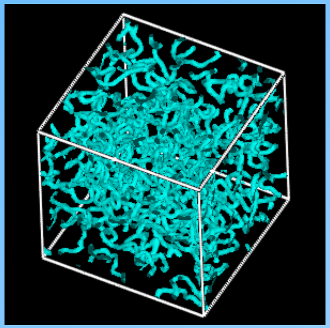

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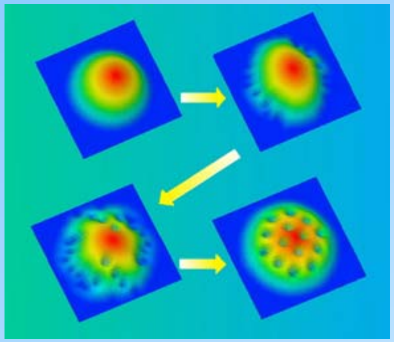

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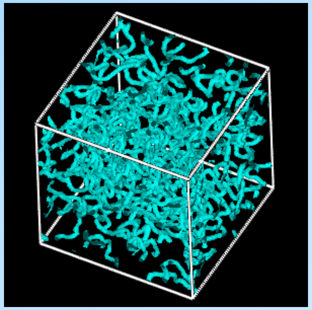

3. Quantized vortices in atomic BECs
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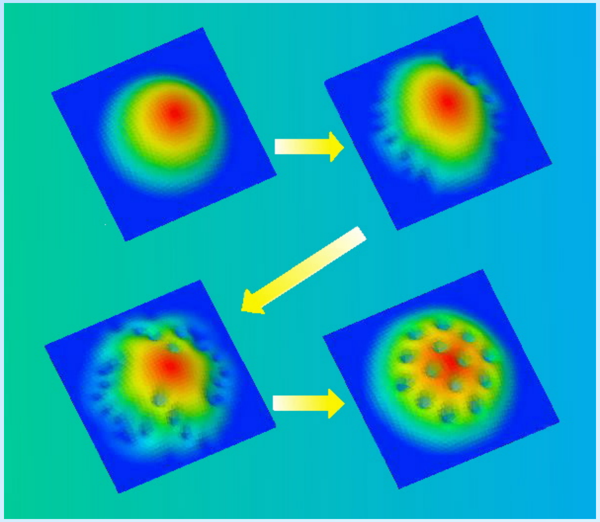


1. Introduction

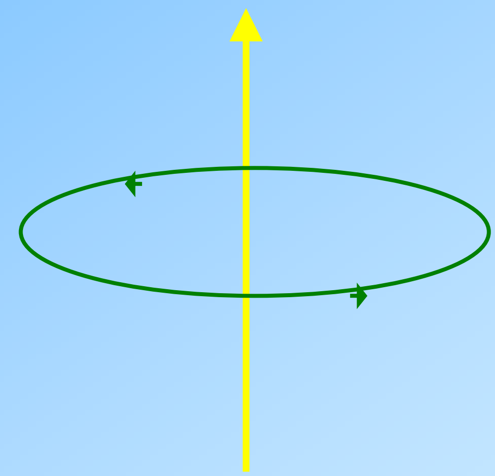
A quantized vortex is a vortex of superflow in a BEC.

(i) The circulation is quantized.

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n \qquad (n = 0, 1, 2, \cdots)$$

$$\kappa = h/m$$

A vortex with $n \geqq 2$ is unstable.



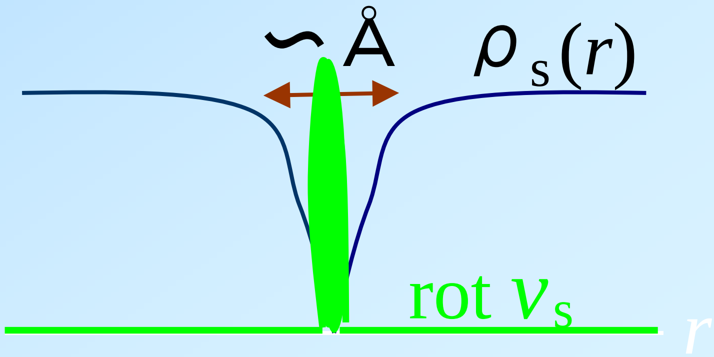
⇨ **Every vortex has the same circulation.**

(ii) Free from the decay mechanism of the viscous diffusion of the vorticity.

⇨ **The vortex is stable.**

(iii) The core size is very small.

⇨ **The order of the coherence length.**



Superfluid helium and the two-fluid model

Liquid ^{4}He enters the superfluid state at 2.17K (λ point) with Bose condensation.

Its hydrodynamics is well described by the two-fluid model.

The two-fluid model

The system is a mixture of inviscid superfluid and viscous normal fluid.

$$\rho = \rho_s + \rho_n$$

point

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Temperature

The two-fluid model could explain various phenomena of superfluidity which were observed experimentally.

The superfluidity breaks down when it flows fast.

1955 **Feynman** proposed that "superfluid turbulence" consisting of a tangle of quantized vortices.

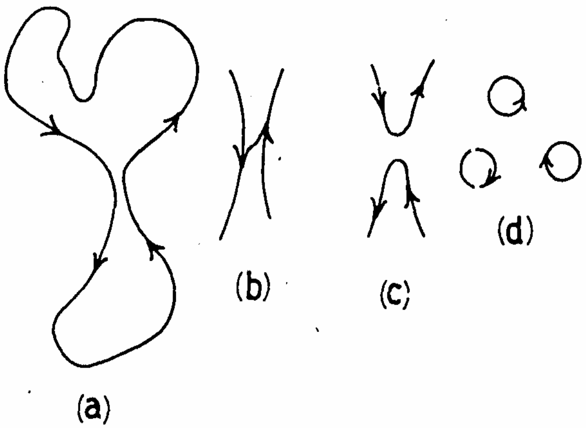
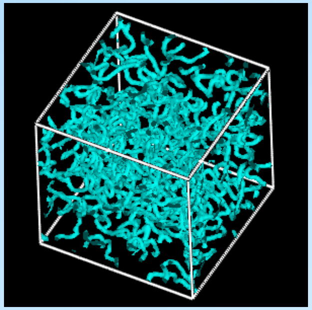


Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

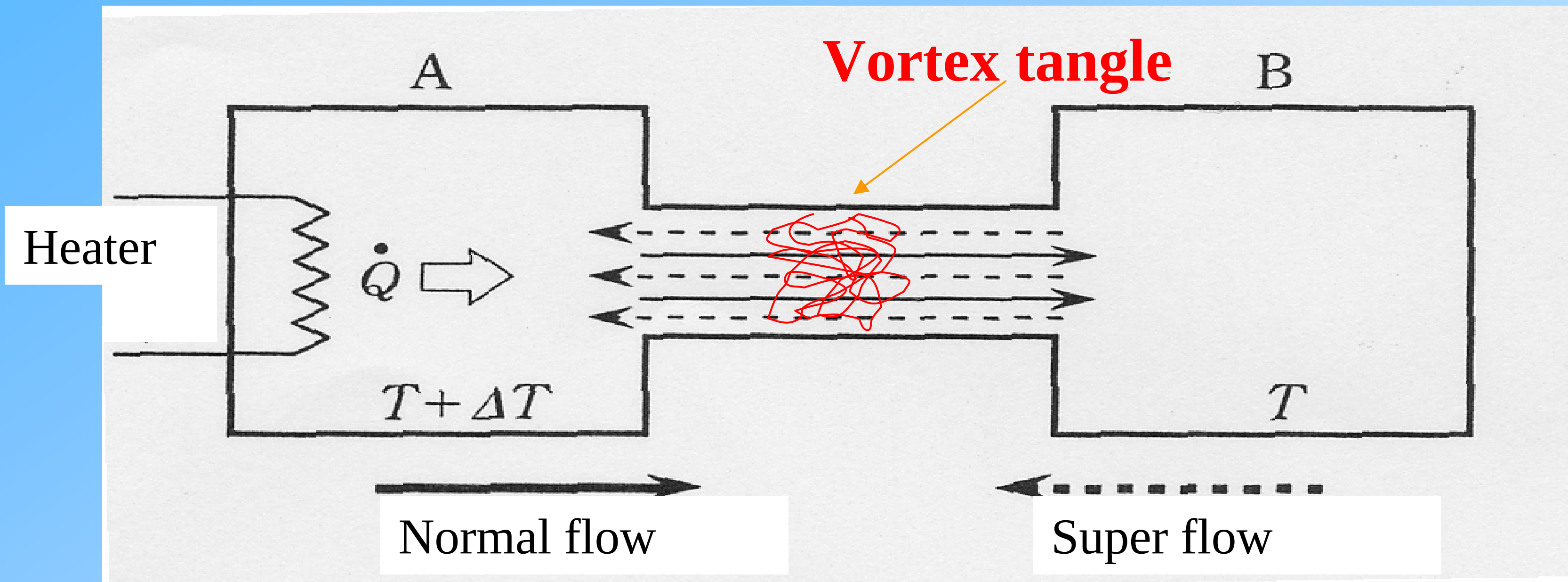
Progress in Low Temperature Physics Vol.I (1955), p.17



1955, 1957 **Hall and Vinen** observed "superfluid turbulence".

The mutual friction between the vortex tangle and the normal fluid causes the dissipation of the flow.

Lots of experimental studies were done chiefly for thermal counterflow of superfluid ^{4}He.



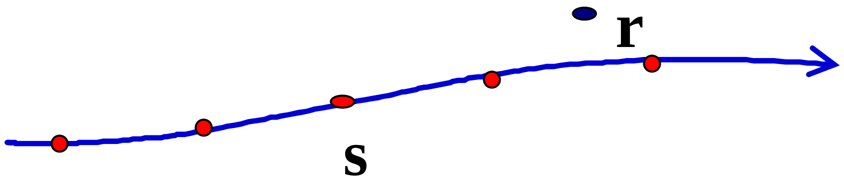
1980's **K. W. Schwarz** Phys.Rev.B38, 2398(1988)

Made the direct numerical simulation of the three-dimensional dynamics of quantized vortices and succeeded in explaining quantitatively the observed temperature difference ΔT .

How to describe the vortex dynamics

Vortex filament formulation (Schwarz) **Biot-Savart law**

$$\mathbf{v}_s(\mathbf{r}) = \frac{\kappa}{4\pi}\int \frac{(\mathbf{s}-\mathbf{r})\times d\,\mathbf{s}}{|\mathbf{s}-\mathbf{r}|^3}$$



A vortex makes the superflow of the Biot-Savart law, and moves with this local flow. At a finite temperature, the mutual friction should be considered. A filament is numerically represented by a string of points.

The Gross-Pitaevskii equation for the macroscopic wave function

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2\right]\Psi(\mathbf{r},t)$$

Development of a vortex tangle in a thermal counterflow

M. Tsubota, T.Araki and S.K. Nemirovskii, Phys. Rev. B62, 11751 (2000)

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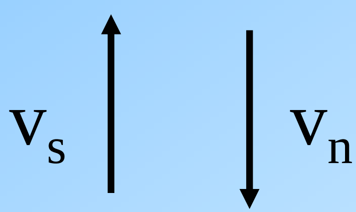

Schwarz, Phys.Rev.B38, 2398(1988).

Schwarz obtained numerically the statistically steady state of a vortex tangle which is sustained by the competition between the applied flow and the mutual friction. The obtained vortex density $L(v_{ns}, T)$ agreed quantitatively with experimental data.

Most studies of superfluid tur
thermal counterflow.

⇨ **No analogy with**

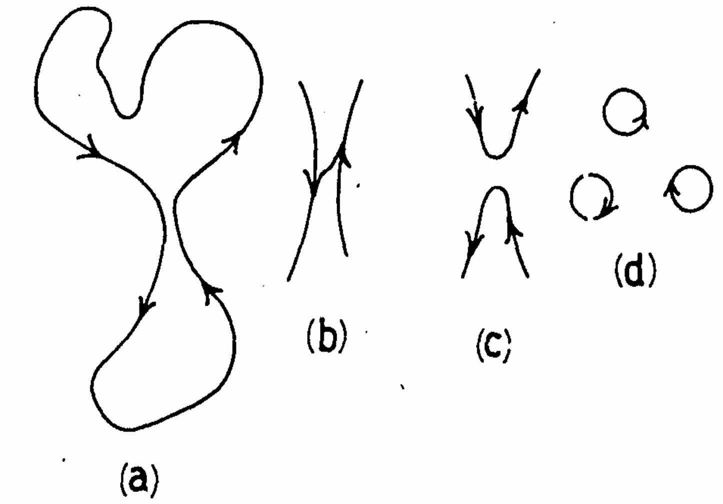

Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

When Feynman showed the above figure, he thought of a cascade process in classical turbulence.

What is the relation between superfluid turbulence and classical turbulence？

Classical turbulence and vortices

Turbulence behind a dragonfly

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Numerical analysis of the Navier-Stokes equation made by Shigeo Kida

Vortex cores are visualized by tracing pressure minimum in the fluid.

Classical turbulence and vortices

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- The vortices have different circulation and different core size.
- The vortices repeatedly appear, diffuse and disappear.

It is difficult to identify each vortex!

Compared with quantized vortices.

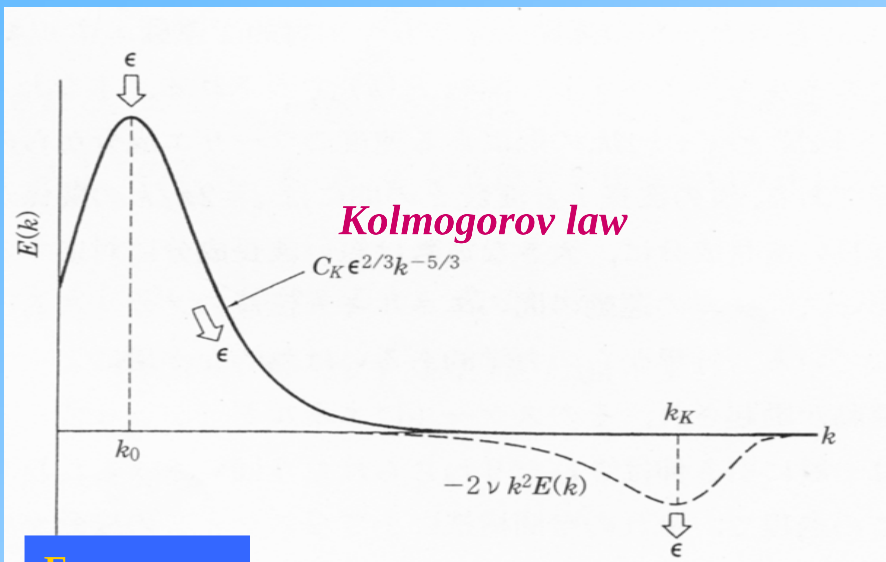
Numerical analysis of the Navier-Stokes equation made by Shigeo Kida

Vortex cores are visualized by tracing pressure minimum in the fluid.

Classical turbulen

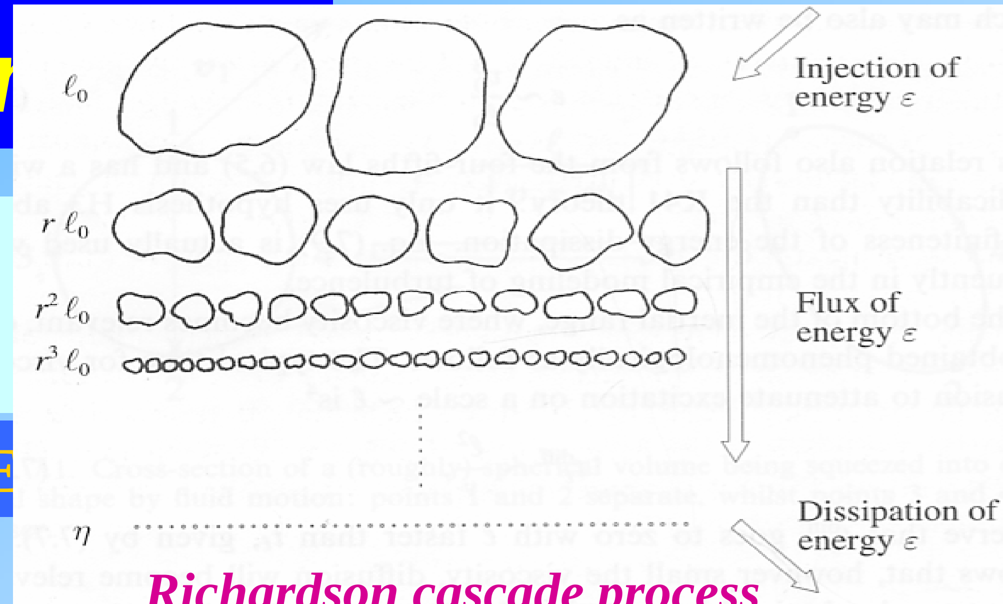
Energy spectrum of the velocity field

$$E = \int \frac{\rho}{2} \mathbf{V}^2 d\mathbf{r} = \int E(k) dk$$



Energy-containing range | Inertial range | Energy-dissipative range

Energy spectrum of turbulence



Richardson cascade process

E

The ..

Inertial range

Dissipation does not work. The nonlinear interaction transfers the energy from low *k* region to high *k* region.

Kolmogorov law : $E(k) = C\,\varepsilon^{2/3} k^{-5/3}$

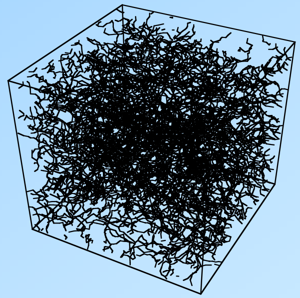
Energy-dissipative range

The energy is dissipated with the rate ε at the Kolmogorov wave number $k_c = (\varepsilon / \nu^3)^{1/4}$.

2. Quantized vortices in superfluid helium

2-1 Recent interests in superfluid turbulence (ST)

ST consists of a tangle of quantized vortices.



Characteristics of quantized vortices

•Quantization of the circulation ・Very thin core

•No viscous diffusion of the vorticity

•A quantized vortex is a stable and definite topological defect, compared with vortices in a classical fluid. The only alive freedom is the topological configuration of its thin cores.

∽Vortex skeletons as elementary vortices ∽

•Because of superfluidity, some dissipation would work only at large wave numbers (at very low temperatures).

Superfluid turbulence may give a prototype of turbulence, much simpler than conventional turbulence.

Can such quantized vortices still produce the essence of turbulence?

The studies of ST have entered a new stage since the middle of 90's !

Experimental confirmation of the Kolmogorov law

Maurer and Tabeling, Europhysics. Letters. 43, 29(1998)

Stalp, Skrbek and Donnely, Phys.Rev.Lett. 82, 4831(1999)

Theoretical consideration

Vinen, Phys.Rev.B61, 1410(2000)

Numerical study of the Kolmogorov law at a finite temperature

Kivotides, Vassilicos, Samuels and Barenghi,

Europhysics Lett. 57, 845(2002)

Review article

Vinen and Niemela, J. Low Temp. Phys. 128, 167(2002) etc.

2-2 Energy spectrum of superfluid turbulence

There are three works which directly study the energy spectrum of ST at zero temperature.

Decaying Kolmogorov turbulence in a model of superflow

C. Nore, M. Abid and M. E. Brachet, Phys.Fluids 9, 2644(1997)

The Gross-Pitaevskii (GP) model

Energy Spectrum of Superfluid Turbulence with No Normal-Fluid Component

T. Araki, M.Tsubota and S. K. Nemirovskii, Phys.Rev.Lett.89, 145301(2002)

The vortex-filament model

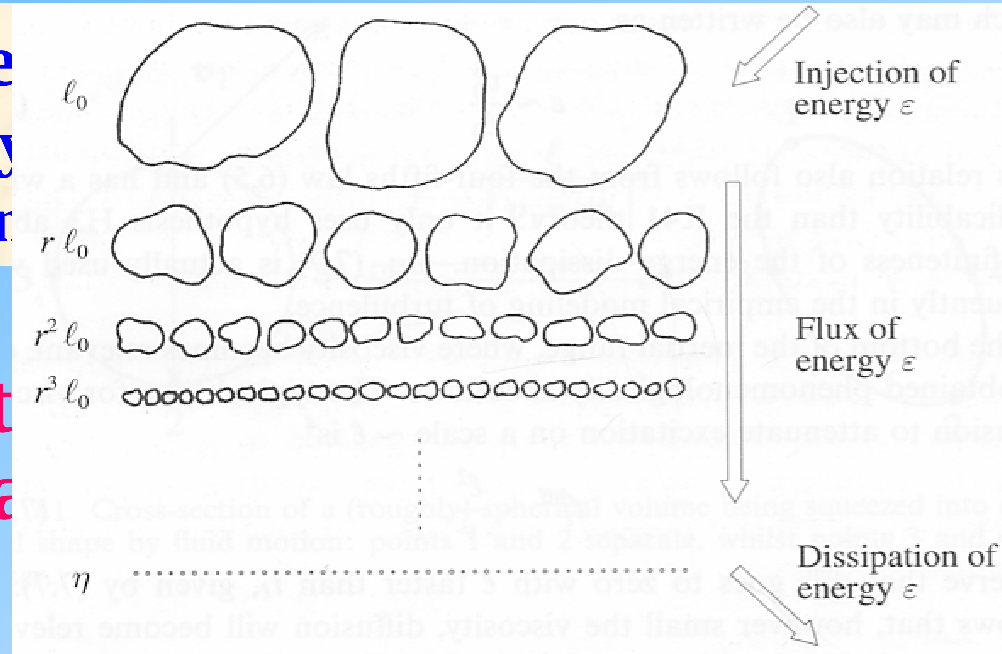
Kolmogorov Spectrum of Superfluid Turbulence: Numerical Analysis of the Gross-Pitaevskii Equation with a Small-Scale Dissipation

M. Kobayashi and M. Tsubota, Phys. Rev. Lett. 94, 065302 (2005) and JPSJ (in press).

Kolmogorov spectrum of supe

M. Kobayashi and M. Tsubota, Phy

J. Phys. Soc. Jpn. (in press) (cond-r



1. We solved the GP equat space in order to use the fa transformation.

2. We made a steady state of turbulence. In order to do that,

2-1 We introduced a dissipative term which dissipates the Fourier component of the high wave number, namely, phonons of short wave length.

2-2 We excited the system at a large scale by moving a random potential.

The GP equation in the Fourier space

$$i\frac{\partial}{\partial t}\Phi(\mathbf{k},t) = \left(k^2 - \mu\right)\Phi(\mathbf{k},t)$$

$$+\frac{g}{V^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Phi(\mathbf{k}_1,t)\Phi^*(\mathbf{k}_2,t)\Phi(\mathbf{k}-\mathbf{k}_1+\mathbf{k}_2,t)$$

$\xi^2 = 1/g|\Phi|^2$ healing length giving the vortex core size

To solve the GP equation numerically with high accuracy, we use the Fourier spectral method in space with the periodic boundary condition in a cube.

How to dissipate the energy at small scales?

The GP equation with the small scale dissipation

$$\{i-\gamma(k)\}\frac{\partial}{\partial t}\Phi(\mathbf{k},t)=\left(k^2-\mu\right)\Phi(\mathbf{k},t)$$

$$+\frac{g}{V^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Phi(\mathbf{k}_1,t)\Phi^*(\mathbf{k}_2,t)\Phi(\mathbf{k}-\mathbf{k}_1+\mathbf{k}_2,t)$$

$\xi^2 = 1/g|\Phi|^2$:healing length giving the vortex core size

$$\gamma(k)=\gamma_0\,\theta(k-2\pi/\xi)$$

We introduce the dissipation that works only in the scale smaller than ξ .

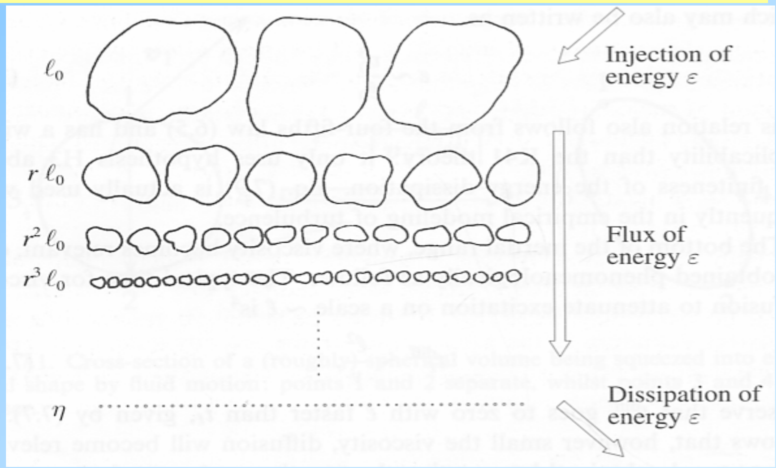
How to inject the energy at large scales?

This is done by moving the random potential satisfying the space-time correlation:

$$\left\langle V(\mathbf{x},t)V(\mathbf{x}',t')\right\rangle = V_0^2 \exp\left[-\frac{(\mathbf{x}-\mathbf{x}')^2}{2X_0^2}-\frac{(t-t')^2}{2T_0^2}\right]$$

The variable X_0 determines the scale of the energy-containing range.

V_0=50, X_0=4 and T_0=6.4 × 10^{-2}



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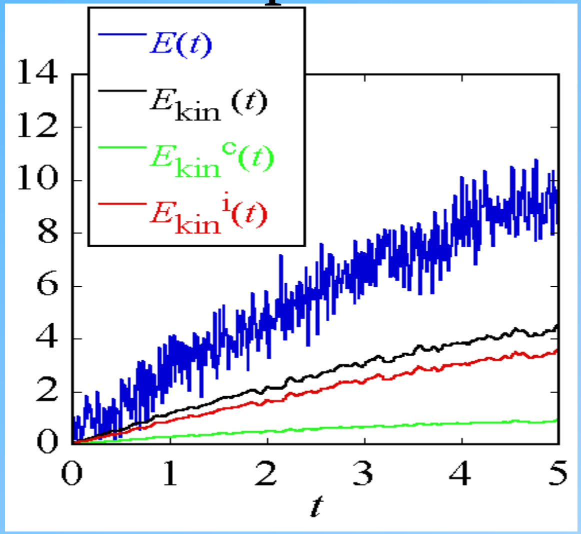
Thus steady turbulence is obtained.(1)

Time development of each energy component



E : Total energy

E_{kin}: Kinetic energy

E_{kin}^{c} : Compressible kinetic energy due to phonons

E_{kin}^{i} : Incompressible kinetic energy due to vortices

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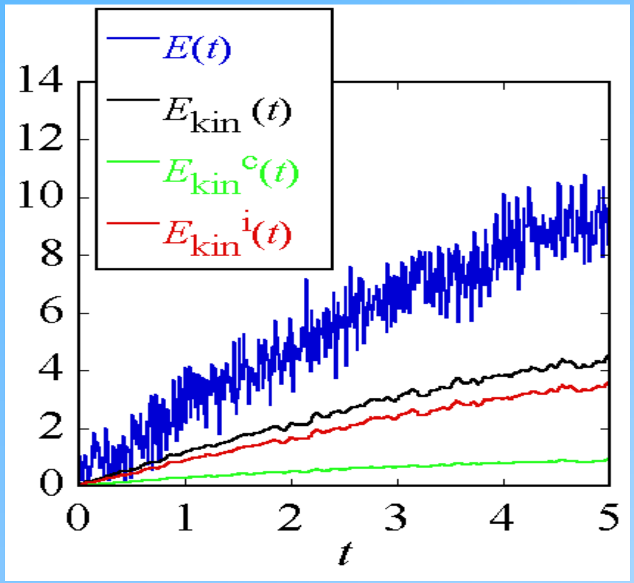
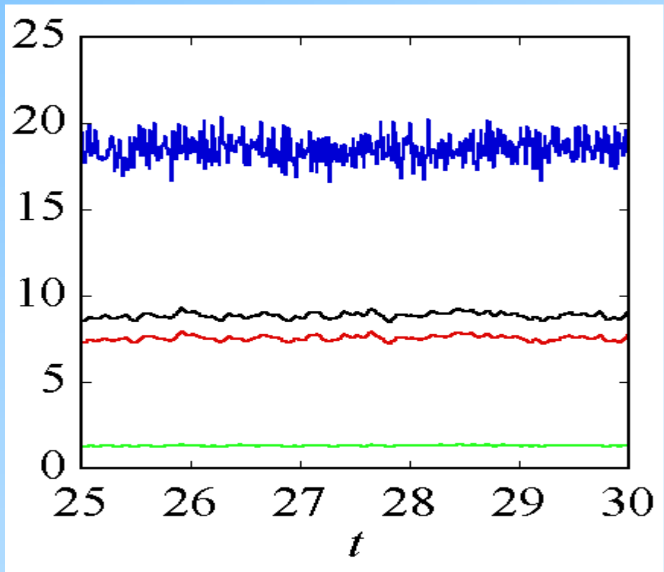
Vortices　　Phase in a central plane　　Moving random potential

Thus steady turbulence is obtained.(2)

Time development of each energy component





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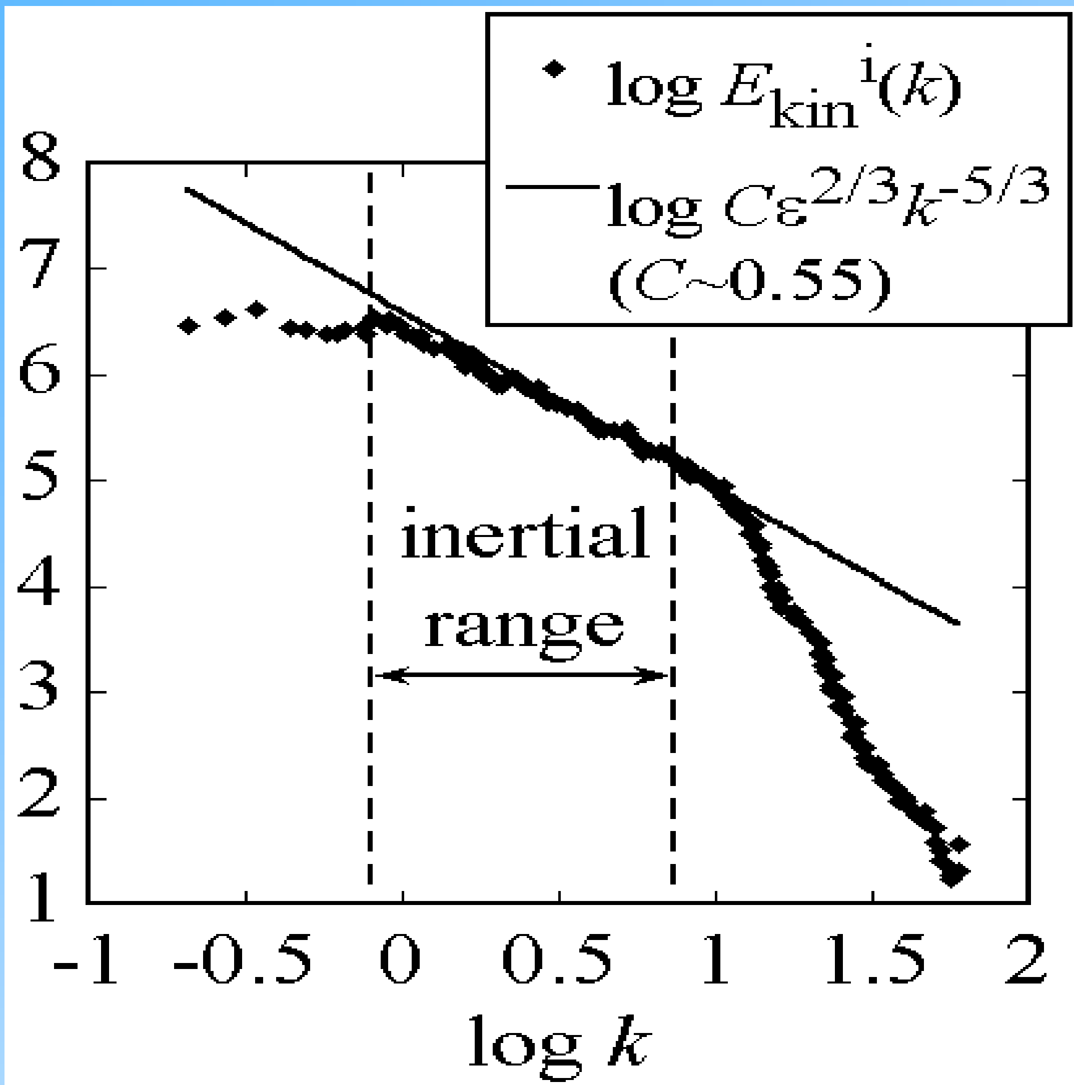
Energy spectrum of the steady turbulence

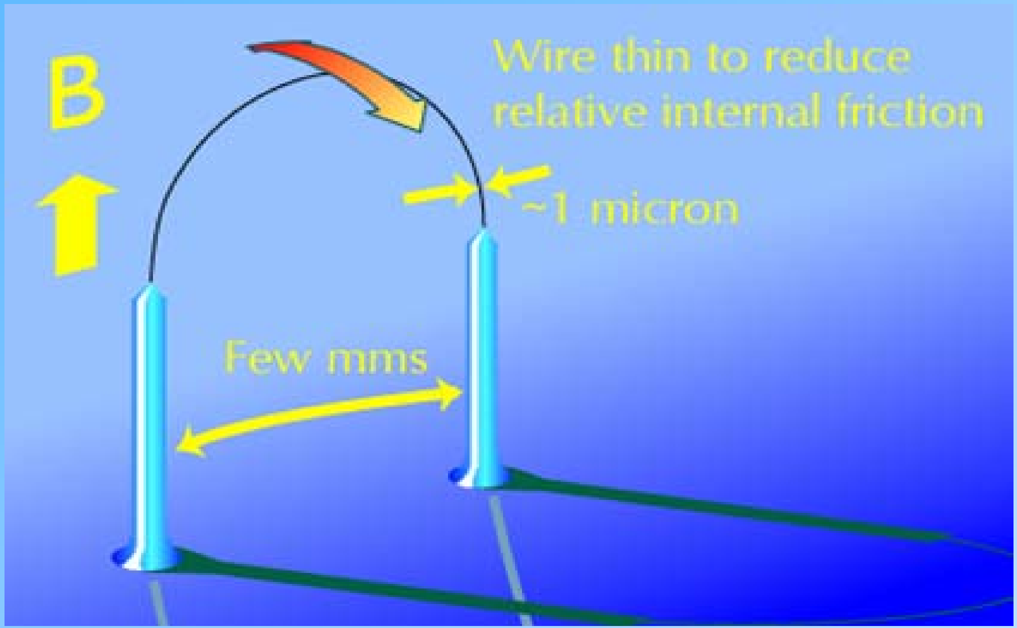


The numerically obtained spectrum is quantitatively consistent with the Kolmogorov law using the dissipation rate ε .

Superfluid turbulence is found to take the essence of classical turbulence!

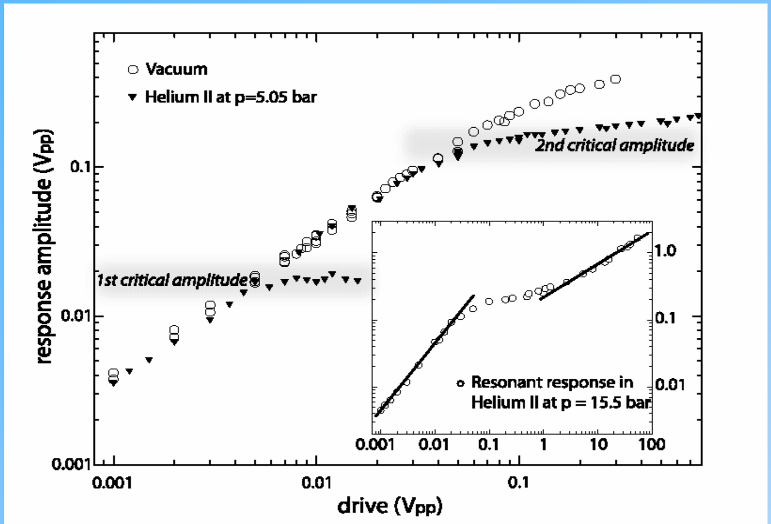
2.3 Turbulence made by a vibrating object

-Vibrating sphere, wire, grid etc.-



We drive such an object in superfluid helium and investigate its response.

Every experiment seems to observe the transition from laminar to turbulent behavior when the driving force is increased.



Lancaster(McClintock, Skrbek *et al.*)

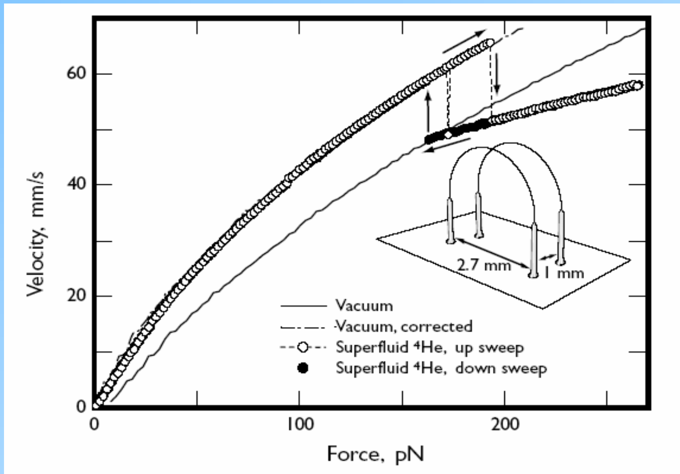
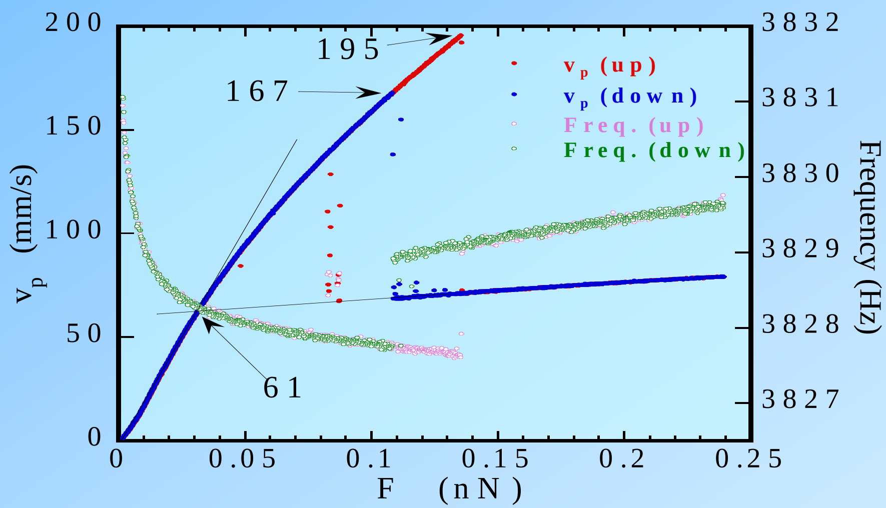


Fig. 1. The velocity response of a wire resonator in superfluid ^{4}He (circles) and in vacuum (solid line). The dashed line shows the vacuum response with a linear damping force subtracted.

Lancaster(Fisher, Pickett *et al.*)



Osaka(Yano, Hata *et al.*)

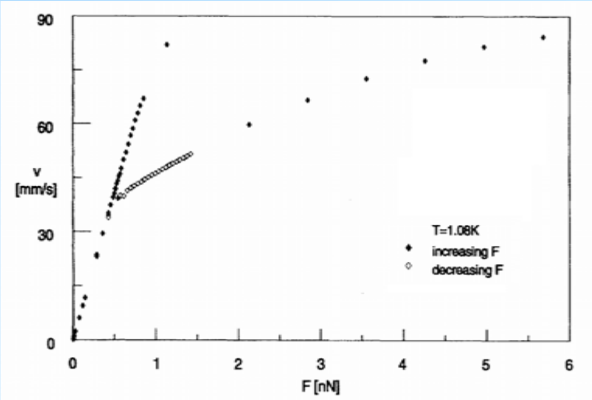


FIG. 4. Transition between laminar and turbulent flow showing the hysteresis for different sweep directions.

Regensburg (Schoepe, *et al.*)

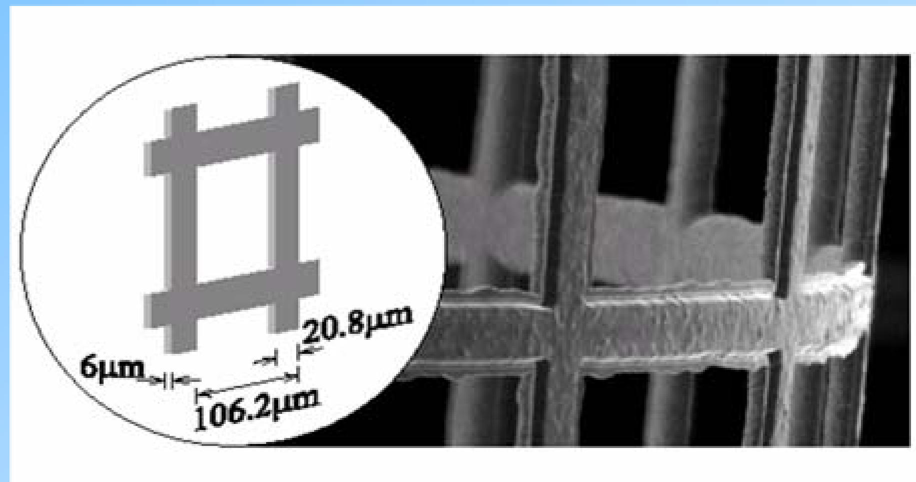
Intrinsic or extrinsic?

The velocity of a wire or a grid is much smaller than the critical velocity for intrinsic vortex nucleation. Hence every observation must come from the dynamics of remnant vortices attached to wires or grids.

However,…..

The surface looks to be very rough. Thus we know clearly neither the initial state of remnant vortices nor the boundary layer.

By Pickett, Lancaster

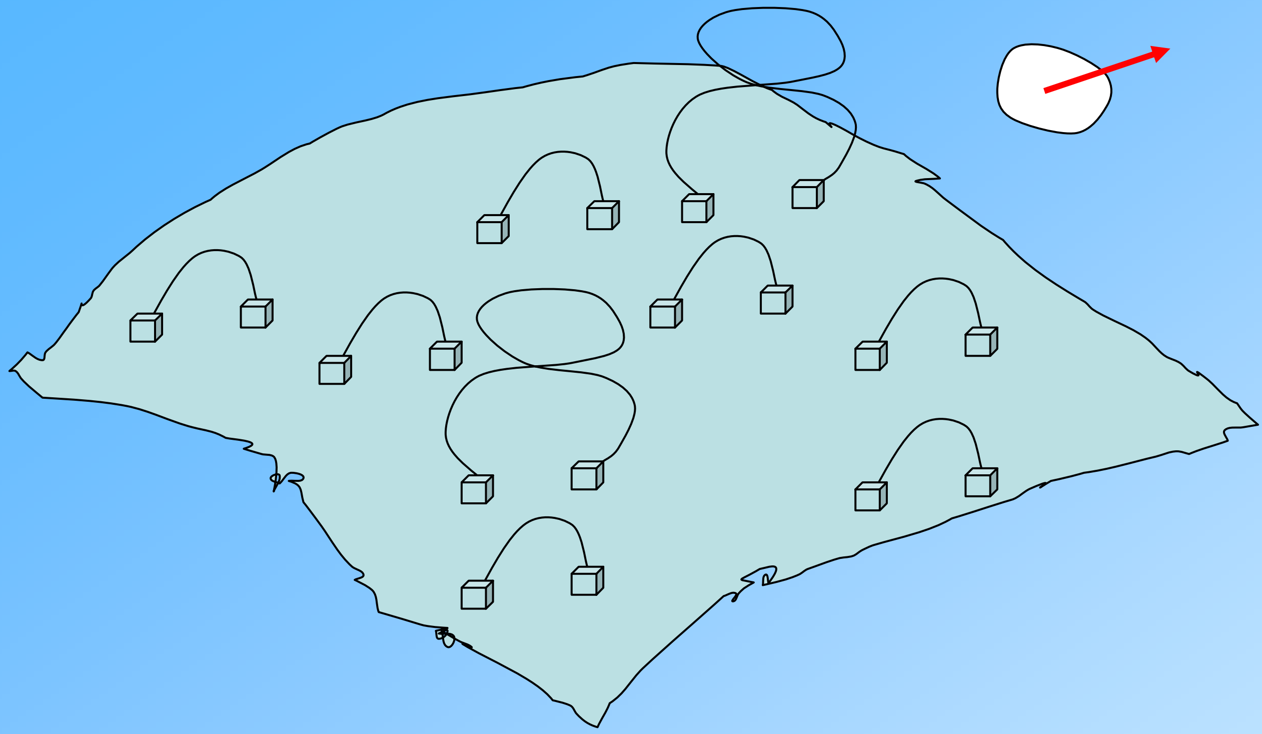


By McClintock, Lancaster

Then, what can we do?

It would be just difficult as the first strategy to do the calculation depending strongly on each geometry and condition.

Boundary layer of pinned vortices subject to AC flow

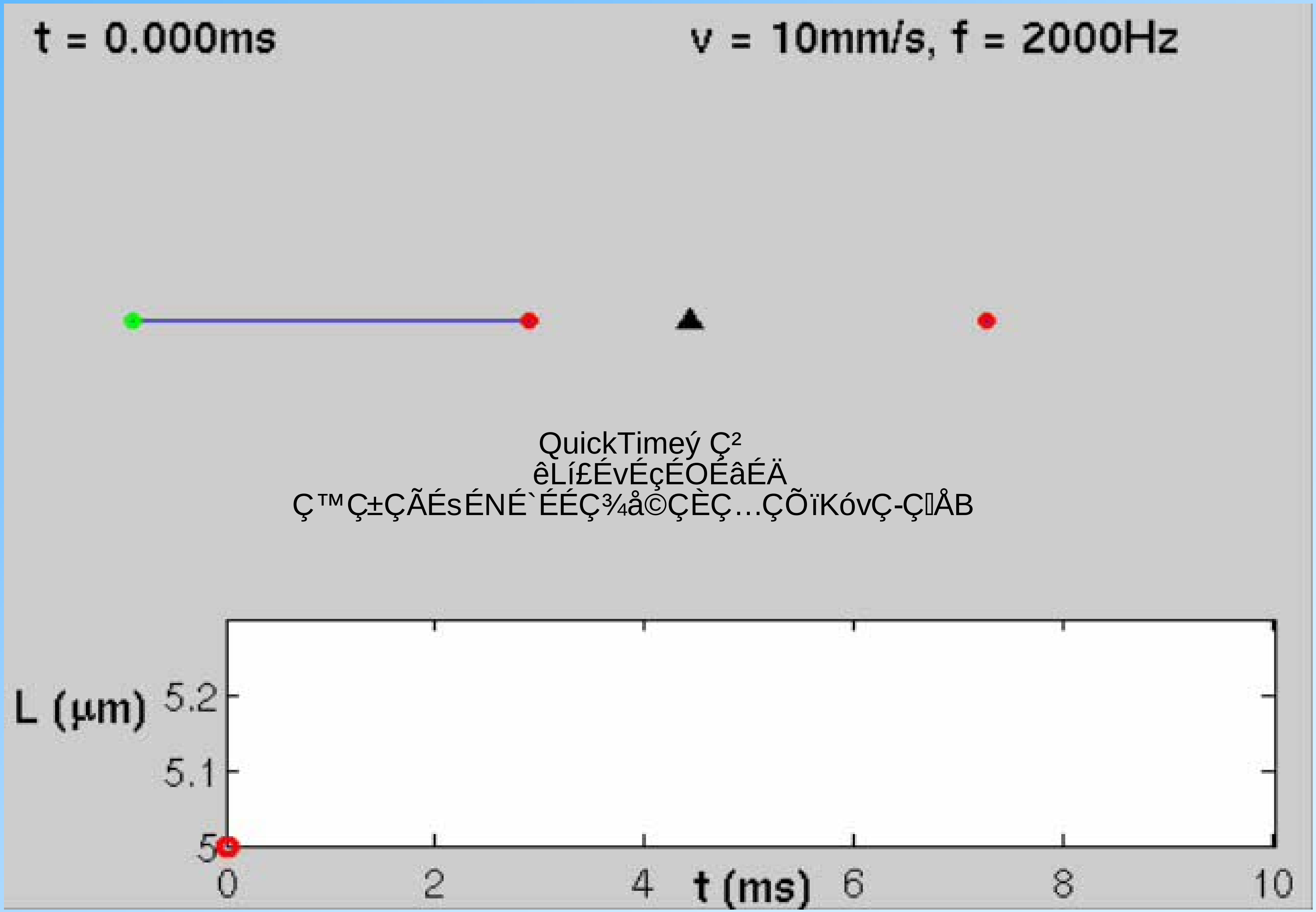


From Skrbek

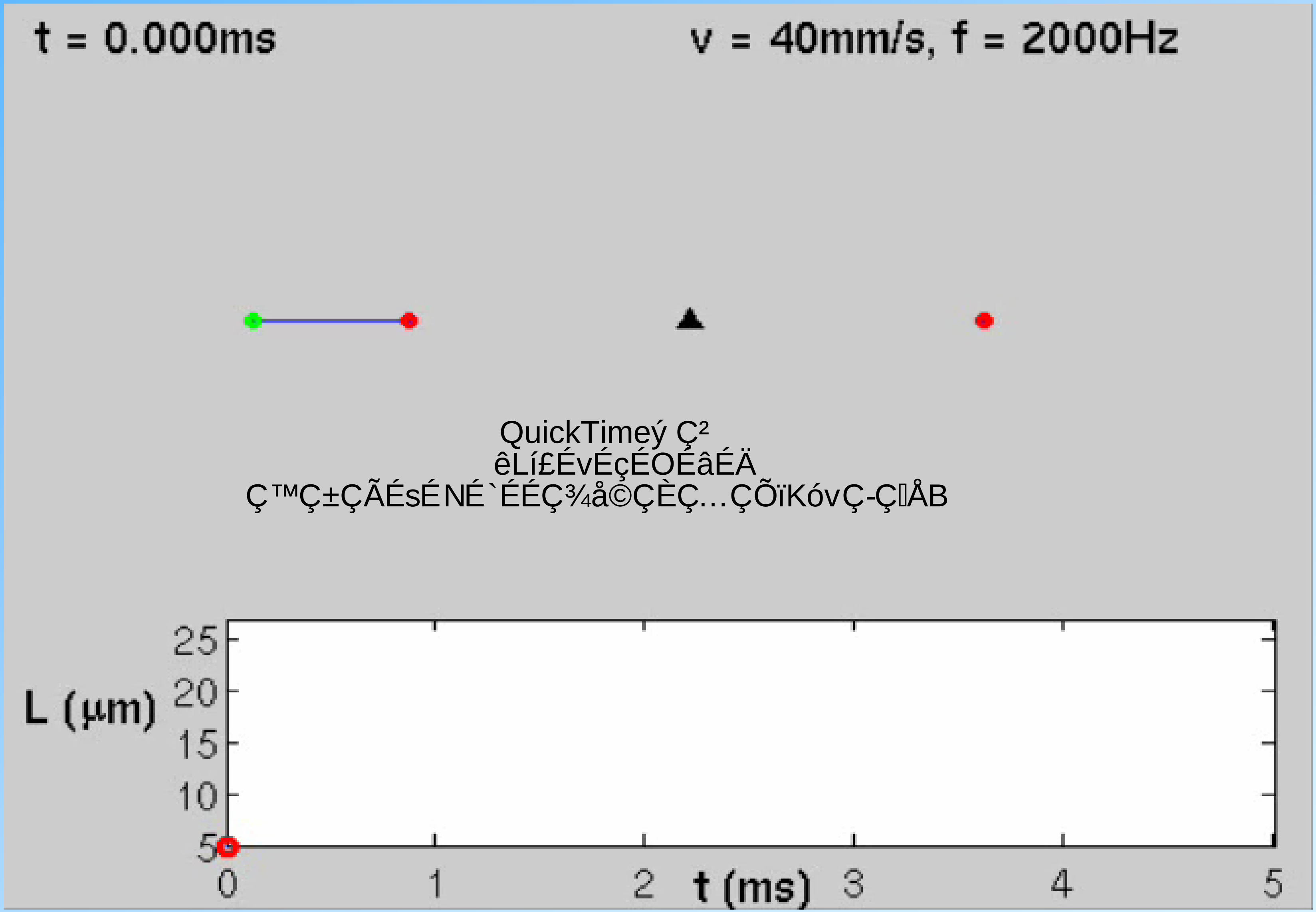
We do not know exactly the configuration of remnant vortices. Our first strategy is to understand the essence of the dynamics of a pinned vortex subject to an AC flow.

Then we reach a simple pinning model.

Results (low driving velocity)



Results (high velocity)



Summary of what happens

- **At low driving velocities, the vortex just oscillates with the frequency resonant with the lowest Kelvin wave.**
- **When the velocities are increased, the vortex motion becomes irregular and the higher modes of Kelvin waves are excited, leading to vortex reconnections and emitting lots of vortex loops.**

The minimum critical velocity is about 20mm/s, mainly determined by the separation of two pinning sites.

Development of a vortex loop attached to a sphere under AC flow

R=100 μ m

V=200mm/s

ω=200Hz

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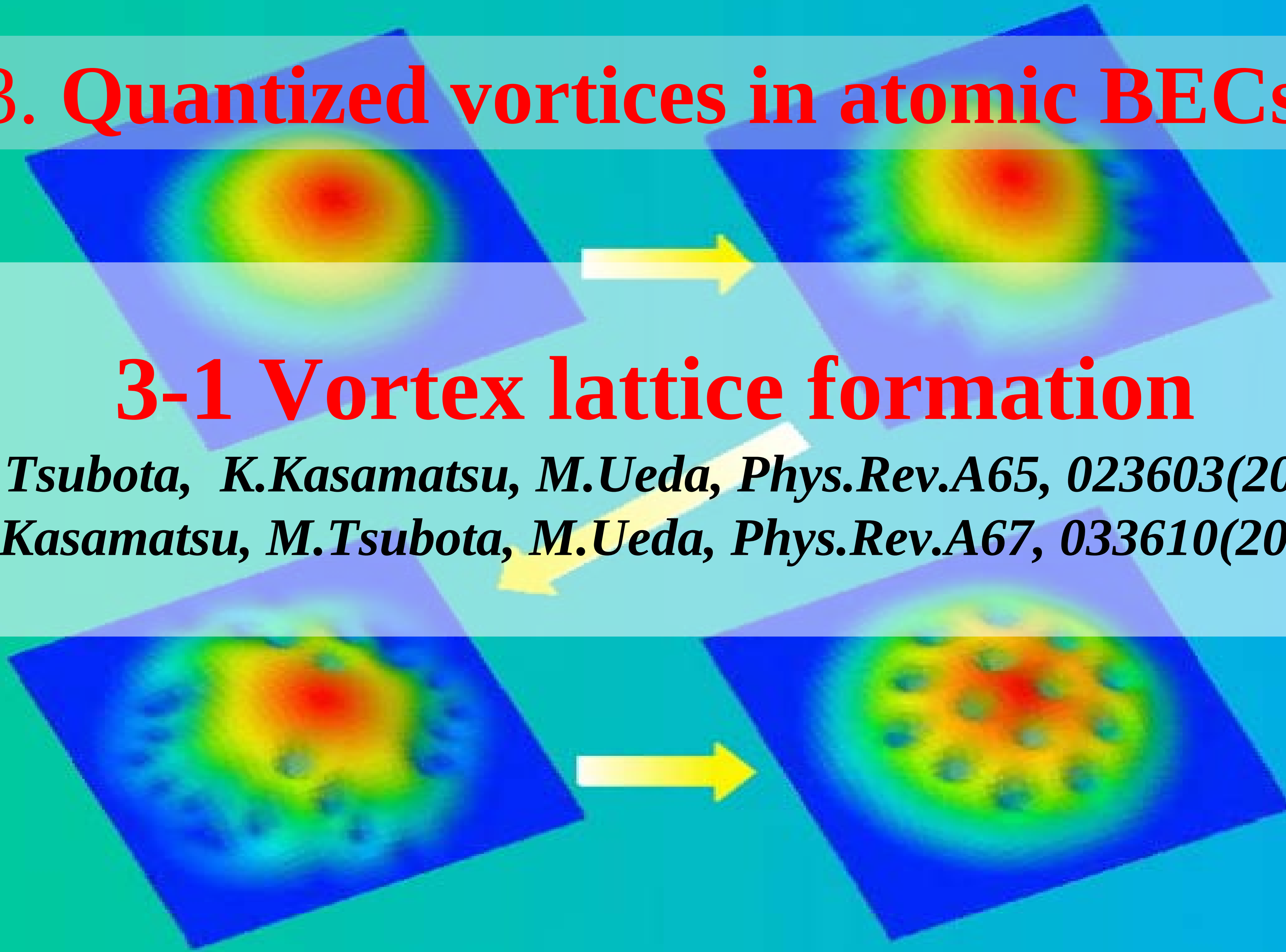
The observed change of the effective mass may be related with this phenomena.

3. Quantized vortices in atomic BECs

3-1 Vortex lattice formation

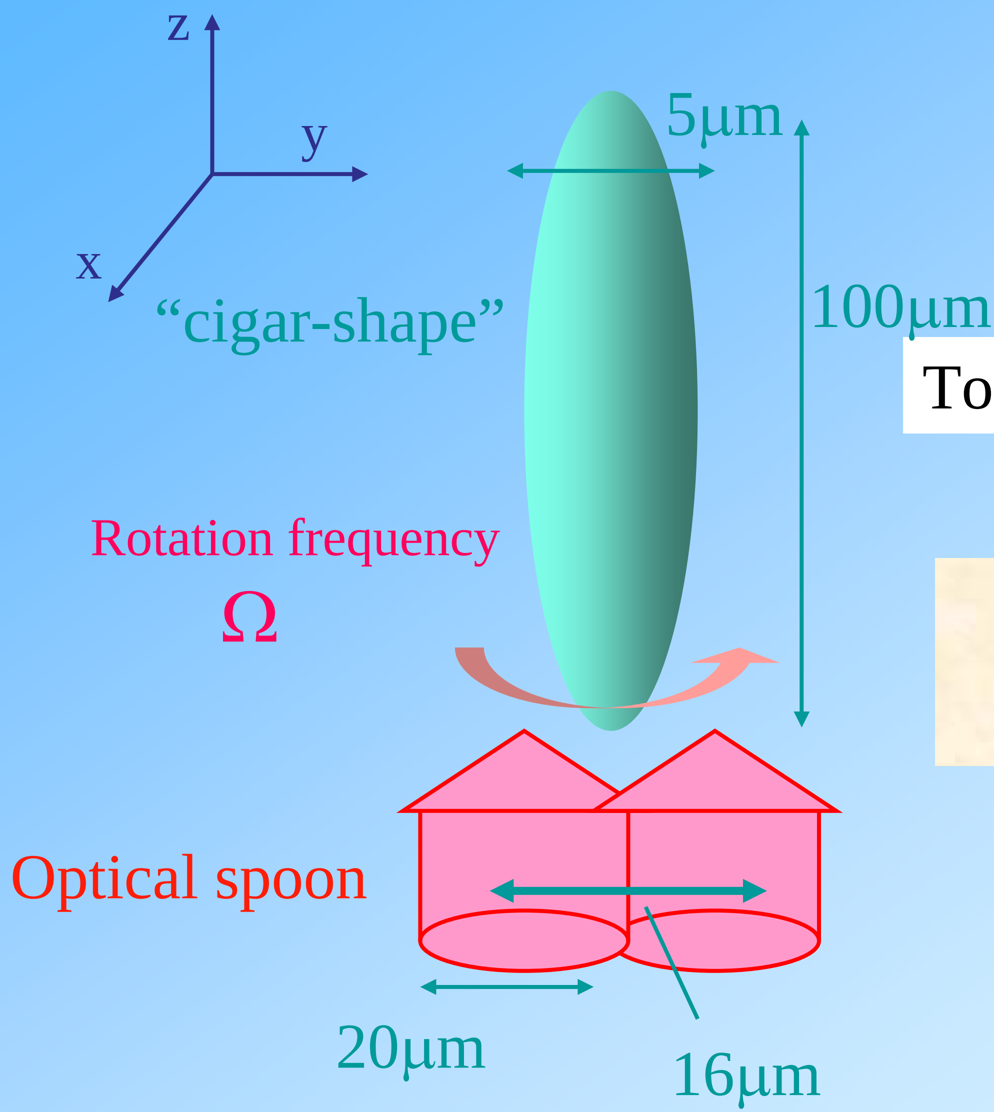
M.Tsubota, K.Kasamatsu, M.Ueda, Phys.Rev.A65, 023603(2002)

K.Kasamatsu, M.Tsubota, M.Ueda, Phys.Rev.A67, 033610(2003)



How can we rotate the trapped BEC？

K.W.Madison et.al Phys.Rev Lett **84**, 806 (2000)



Axisymmetric potential

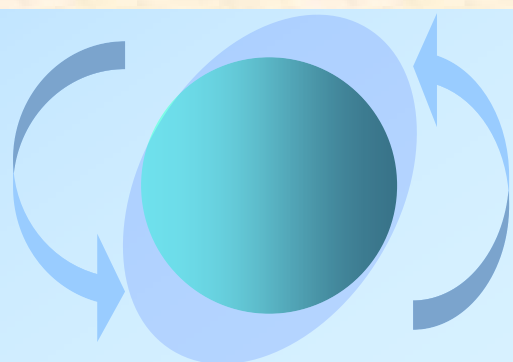
$$V_{\text{ext}}(\mathbf{R}) = V_{\text{trap}}(\mathbf{R}) + U_{\text{stir}}(\mathbf{R})$$

Total potential

Non-axisymmetric potential

$$U_{\text{stir}}(\mathbf{R}) = \frac{m}{2}\omega_{\perp}^{2}(\varepsilon_x X^2 + \varepsilon_y Y^2)$$

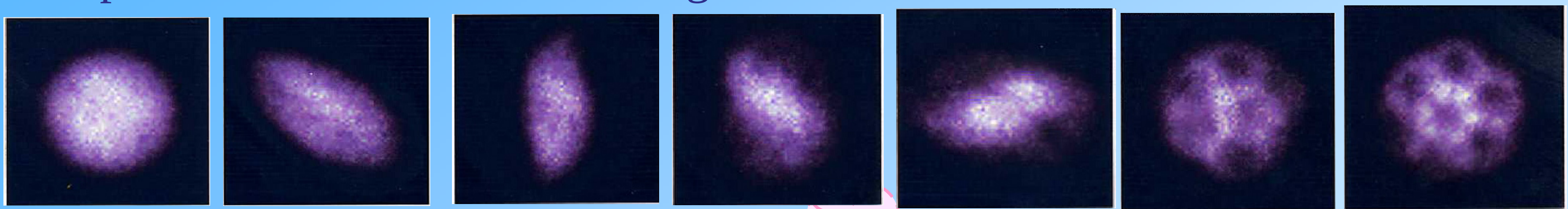
$$\varepsilon_x \neq \varepsilon_y$$



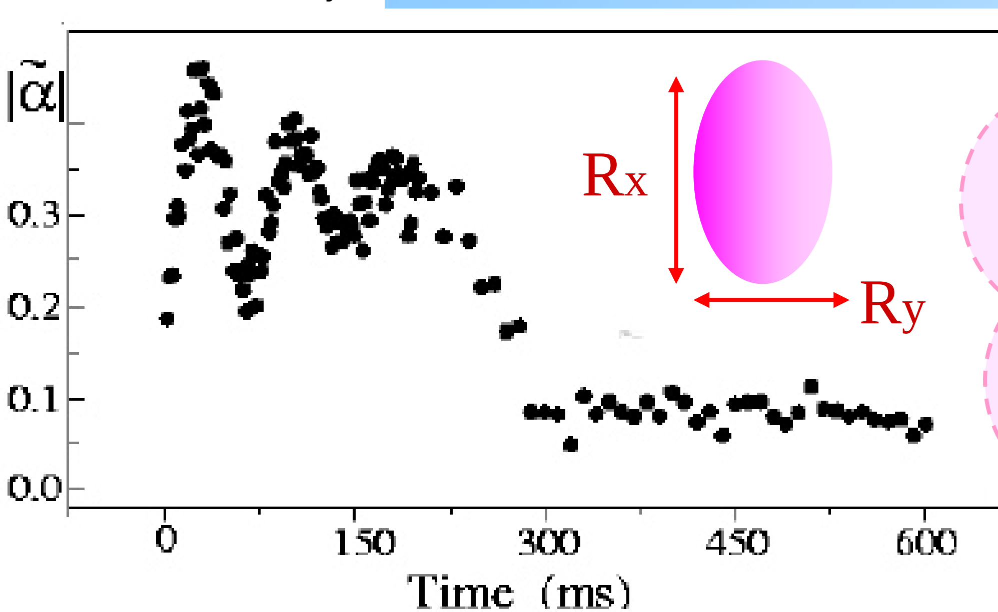
Direct observation of the vortex lattice formation

K.W.Madison et.al.   PRL **86** , 4443 (2001)

Snapshots of the BEC after turning on the rotation



$$\alpha = \Omega \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2}$$



1. The BEC becomes elliptic, then oscillating.
2. The surface becomes unstable.
3. Vortices enter the BEC from the surface.
4. The BEC recovers the axisymmetry, the vortices forming a lattice.

The Gross-Pitaevskii(GP) equation in a rotating frame

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V_{\mathrm{trap}}\Psi + g|\Psi|^2\Psi$$

Wave function $\Psi(\mathbf{r}, t)$

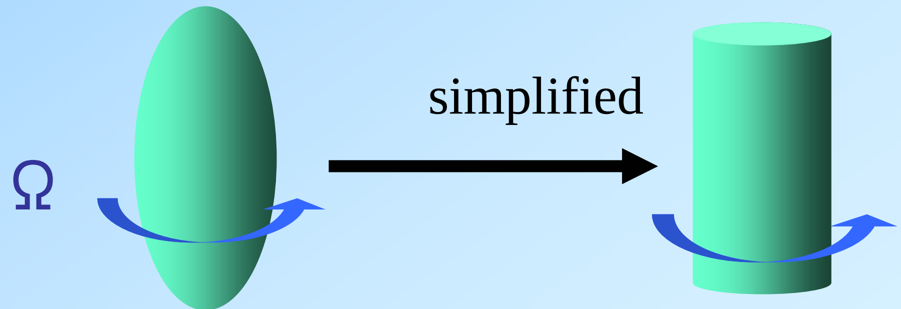
Interaction $g = \frac{4\pi\hbar^2 a_s}{m}$

a_s s-wave scattering length

in a rotating frame

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + (V_{\mathrm{trap}} + U_{\mathrm{stir}})\Psi + g|\Psi|^2\Psi - \Omega L_z\Psi$$

Two-dimensional $U_{\mathrm{stir}}(\mathbf{R}) = \frac{m}{2}\omega_\perp^2(\varepsilon_x X^2 + \varepsilon_y Y^2)$



The GP equation with a dissipative term

$$i\hbar\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+(V_{\mathrm{trap}}+U_{\mathrm{stir}})+g|\Psi|^2-\mu-\Omega L_z\right]\Psi$$

$$(i-\gamma)\hbar\frac{\partial\Psi}{\partial t}$$

$\gamma=0.03$: dimensionless parameter

S.Choi, et.al. PRA 57, 4057 (1998)

I.Aranson, et.al. PRB 54, 13072 (1996)

This dissipation comes from the interaction between the condensate and the noncondensate.

E.Zaremba, T. Nikuni, and A. Griffin, J. Low Temp. Phys. **116**, 277 (1999)

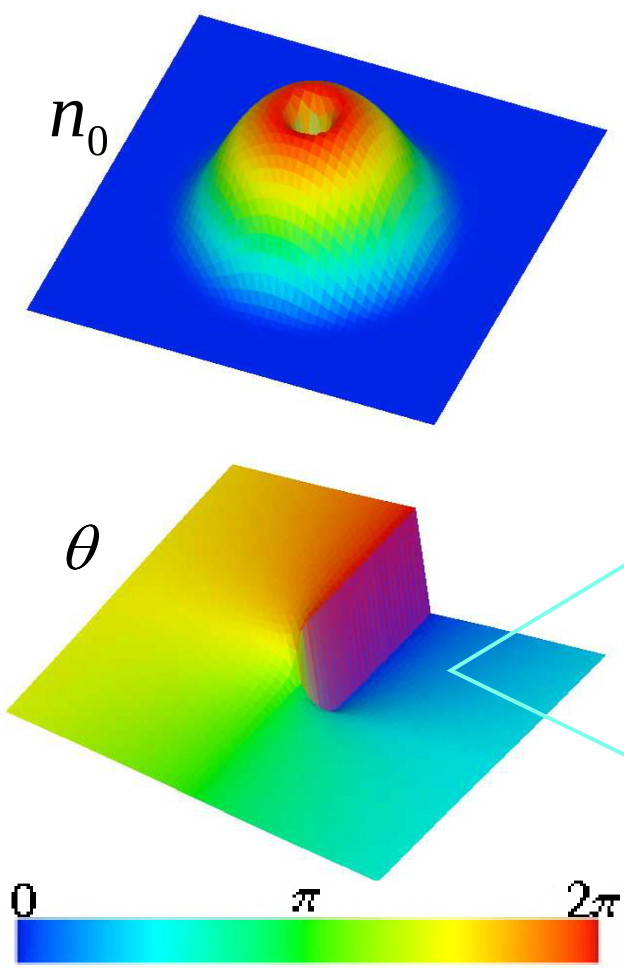
C.W. Gardiner, J.R. Anglin, and T.I.A. Fudge, J. Phys. B **35**, 1555 (2002)

Profile of a single quantized vortex

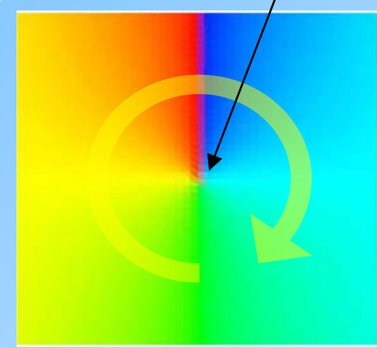
$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V_{\text{trap}}\Psi + g|\Psi|^2\Psi = \mu\Psi$$

$$\Psi(r) = \sqrt{n_0(r)}e^{i\theta(r)}$$

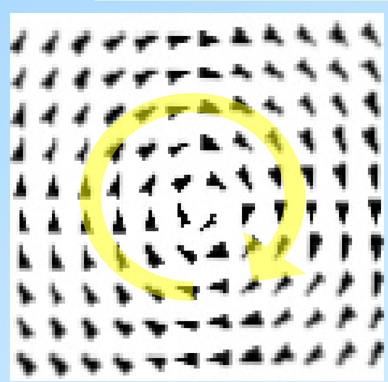
A quantized vortex

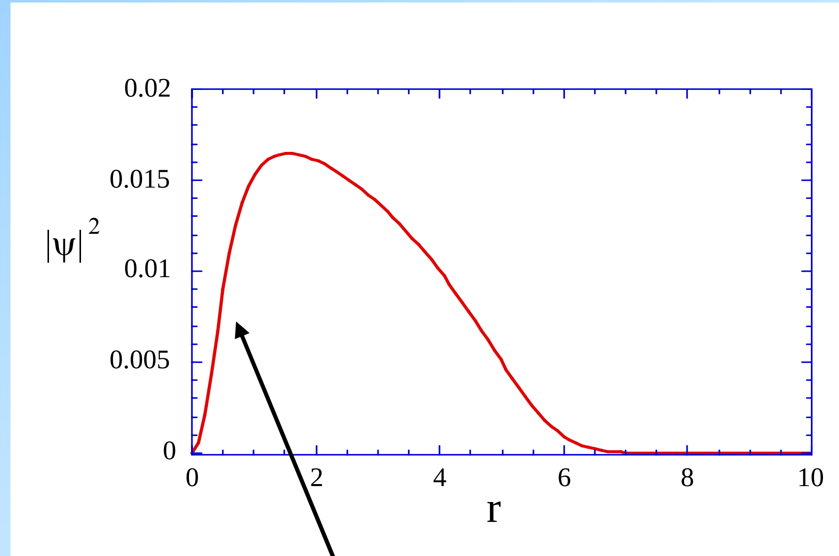


A vortex



Velocity field





Vortex core= healing length

$$\xi \approx \frac{\hbar}{\sqrt{2mgn_0}}$$

Dynamics of the vortex lattice formation (1)

Time development of the condensate density n_0

$$\Omega = 0.7\omega_\perp$$

Tsubota *et al.*,
Phys. Rev. A **65**,
023603 (2002)

$$V_{\text{trap}}(r) = \frac{1}{2} m\omega_\perp^2 r^2$$

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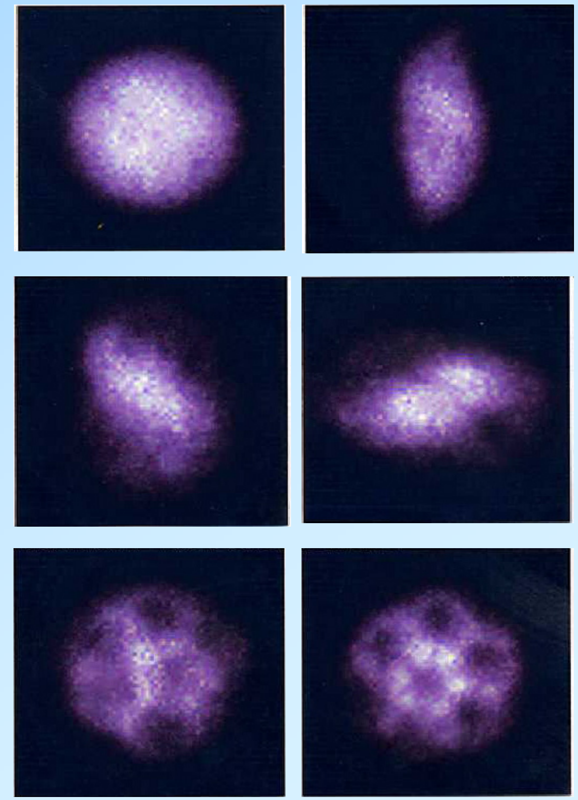
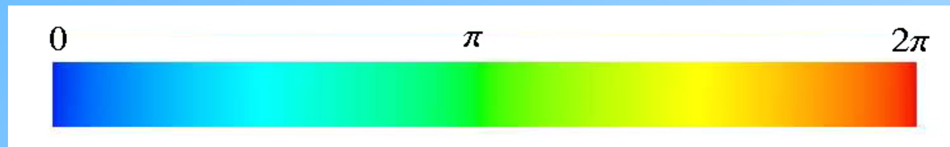
$$\Psi(r) = \sqrt{n_0(r)}\, e^{i\theta(r)}$$

Experiment



Dynamics of the vortex lattice formation (3)

Time-development of the phase θ



$$\Psi(r) = \sqrt{n_0(r)}\,e^{i\theta(r)}$$

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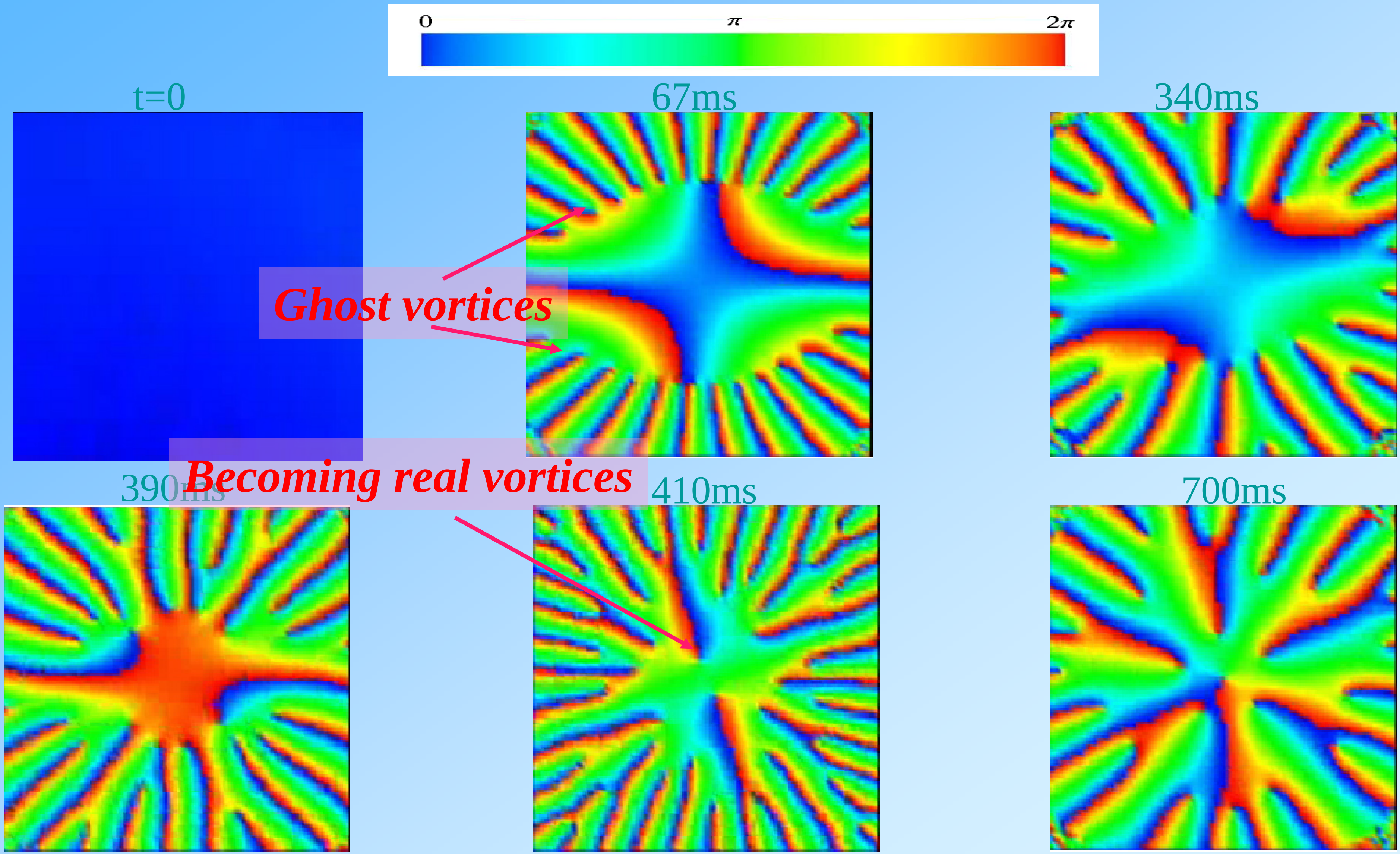
Dynamics of the vortex lattice formation (4)

Time-development of the phase θ



Simultaneous display of the density and the phase

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4. Research plans on this project

Establishment of quantum fluid dynamics

- Quantum turbulence as a prototype of turbulence
- Superfluid turbulence neat $T = 0$ K
- AC superfluid turbulence by a vibrating object

BEC and superfluidity in periodic, random or confined potential

- BEC and superfluidity in an optical (periodic) lattice
- BEC and superfluidity in a nano-porous media

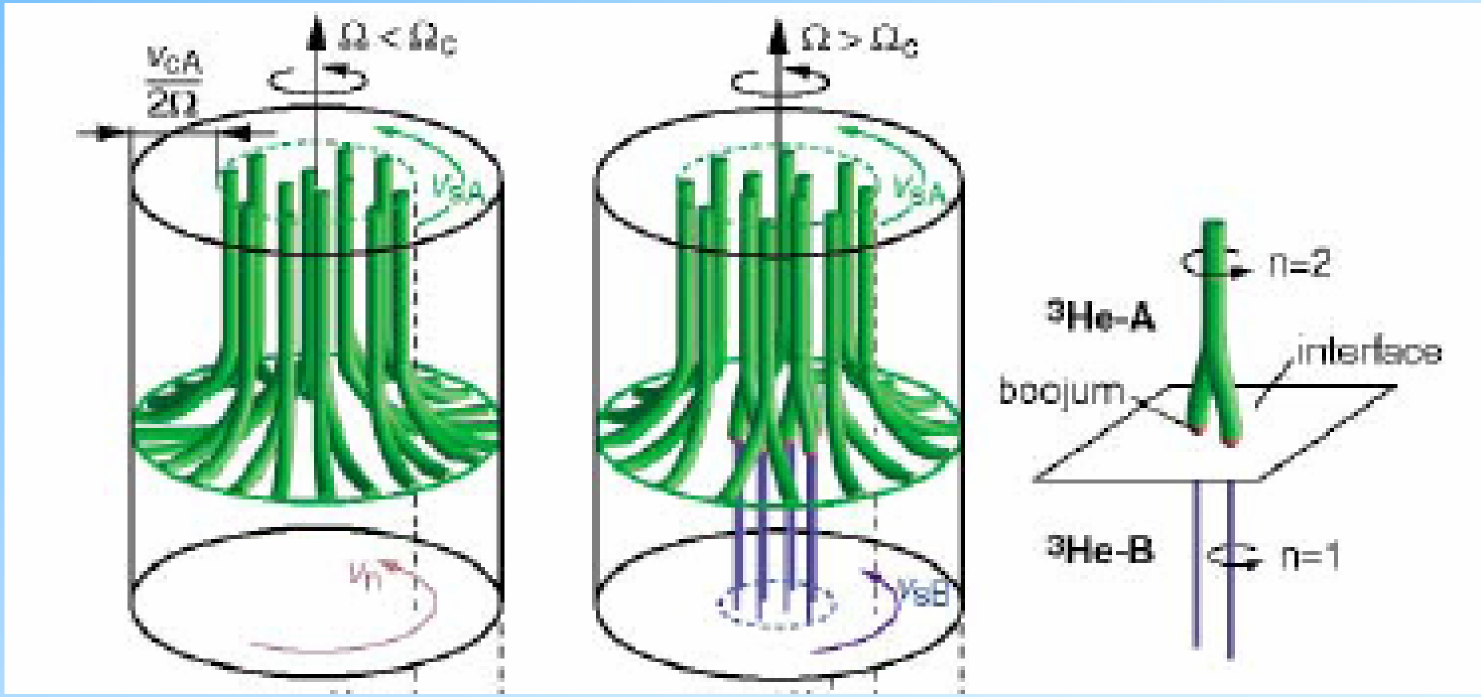
⇨**A02**

Making exotic topological defects in multi-component BECs

⇨**A04**

Studies of superfluid ^{3}He have revealed many kinds of topological defects. Let's make such exotic topological defects in multi-component BECs.

Boojums appear in the interface between ^{3}He-A and -B.



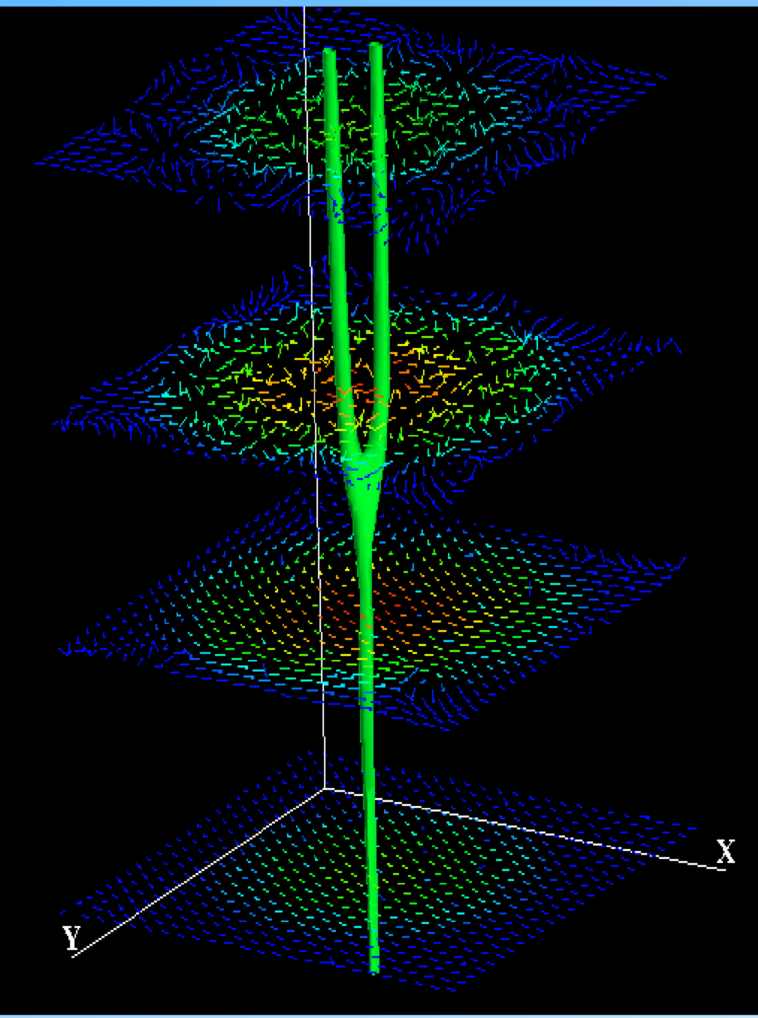
R. Blaauwgeers *et al.*, Phys. Rev. Lett. **89**, 155301(2002)

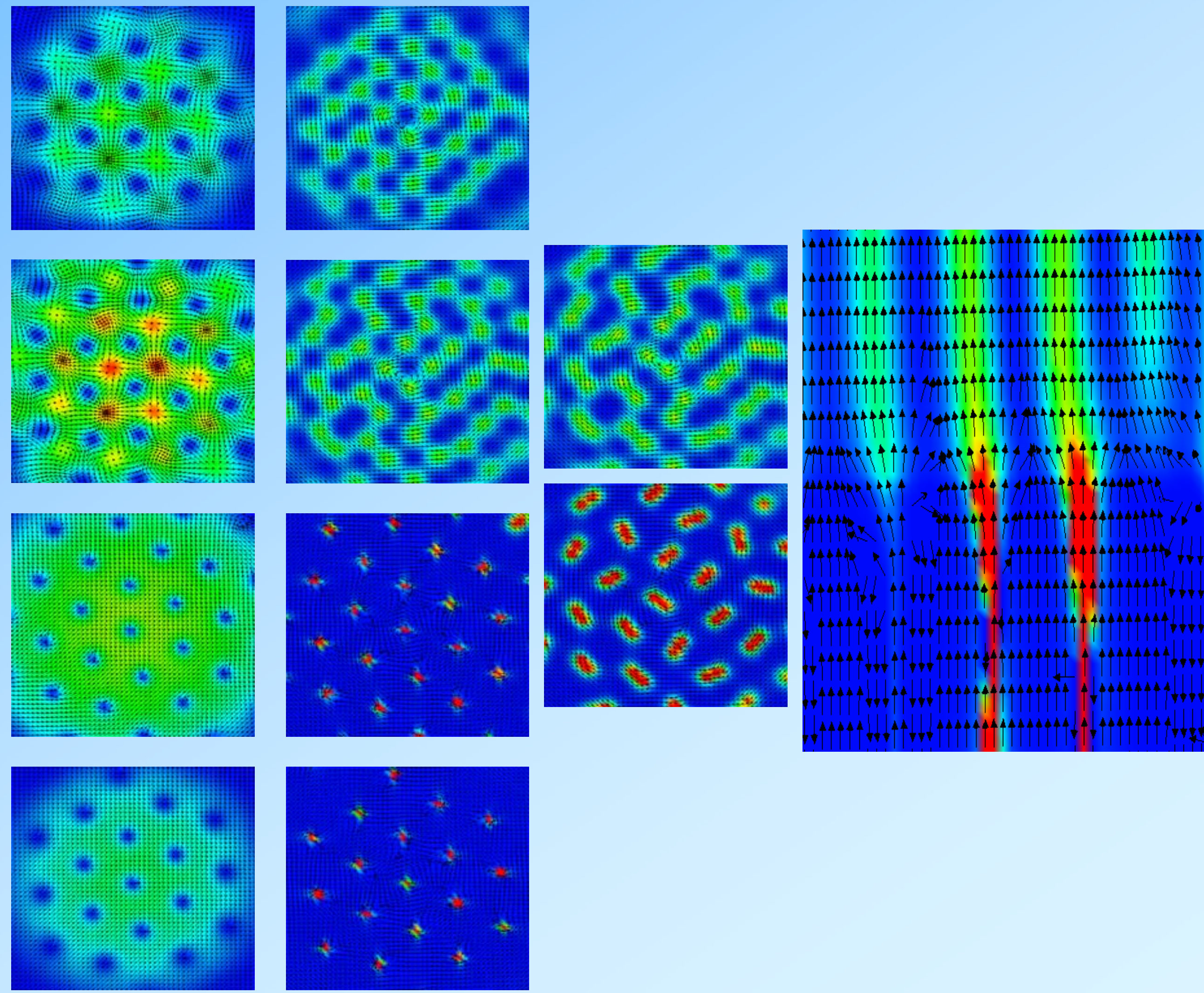
Boojums in two-component BECs

H. Takeuchi and M. Tsubota

Pseudo spin

Vorticity of superflow

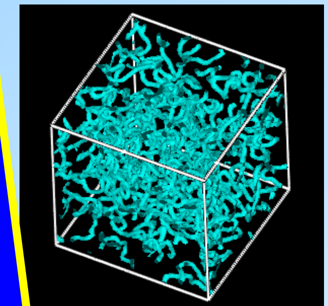




Outline

1. Introduction

2. Quantized vortices in superfluid helium
   2-1 Recent interests in superfluid turbul
   2-2 Energy spectru
   2-



All these topics can be studied only in super clean systems!

3. Quantized vortices in atomic BECs
   3-1 Vortex lattice formation

4. Research plans on this project